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Statistical process control techniques for correlated processes

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ABSTRACT

Control charts used for process control are usually developed under the assumption that the sequence of observations to which they are applied is uncorrelated. This assumption is however frequently violated in practice especially in the continous process industry where the observations of an in control process are correlated. Under such circumstances traditional control charting procedures cannot be used effectively.

A solution to this problem is to model the correlated process as a time series model and use that model to remove the autocorrelation from the data and plot the residuals obtained by back calculation on a control chart. The performance exhibited by a control chart plotted with residuals is compared to a control chart where the correlated observations were plotted. Such an approach was found to give good results under certain circumstances.

An alternate solution to the problem was developed based on the results of the first approach. This approach involved transformation of the residuals obtained from one time series model to another time series model and then plotting the transformed data on a control chart. This approach was found to be effective under a different set of environments.

A combination of the two approaches can lead to effective monitoring and control of a process whose output is serially correlated.

1.0 INTRODUCTION :

Statistical Process Control (SPC) can be generally defined as the use of statistical methods and techniques to improve process productivity and product quality. Included under statistical process control are all the statistical techniques involved with the design of experiments , analysis of data and on - line quality control methods.This thesis is concerned with control chart methods aimed at process monitoring and control for continous processes as would be found in the chemical industry. Due to the natural correlation that exists in such processes there is a need for new control chart methods. This is the purpose of this thesis.

1.1.0 Causes of Variation in Quality:

At the basis of the theory of control charts is a differentiation of the causes of variation in quality. In any production process regardless of how well designed, a certain amount of inherent or natural variability will always exist. This natural variability or background noise is the cumulative effect of many small and essentially uncontrollable causes. When the background noise of a process is relatively small , we usually consider this an acceptable level of process performance. A process that is operating with only chance causes of variation present is said to be in statistical control.

Other kinds of variability may occasionally be present in the output of a process. This variability in the key quality characteristics usually arises from improperly adjusted processes, human error , defective materials. Such variability is generally large when compared to the background noise and usually represents an unacceptable level of process performance. These are refered to as assignable causes. A process that is operating in the presence of assignable causes is said to be out of control.

It is very typical of production processes to operate in an in control state producing acceptable product for relatively long periods of time . Occasionally assignable causes will occur resulting in a shift to an out of control state where a large proportion of the process output does not conform to specifications. A major objective of statistical quality control is to quickly detect the occurence of assignable causes or process shifts so that investigation of the process and corrective action may be taken before many nonconforming units are manufactured. The control charts are an on-line process control technique widely used for this purpose. Control charts are used to estimate the parameters of a production process and the process capability. The control charts provide useful information in improving the process.

1.2.0 Statistical Basis of Control Charts:

The control charts contain a center line that represents the average value of the quality characteristic corresponding to the in control state. Two other lines called the upper control limit (UCL) and the lower control limit (LCL) are chosen such that if the process is in control , nearly all of the sample points will fall between them. As long as the points plot within the control limits , the process is assumed to be in control and no action is necessary. However, a point that plots outside the control limits is interpreted as evidence that the process is out of control , investigative and correction action is required to find and eliminate the assignable causes.

There is a close connection between control charts and hypothesis testing. Essentially, the control chart is a test of the hypothesis that the process is in a state of statistical control. A point plotting within the control limits is equivalent to failing to reject the hypothesis of statistical control and a point plotting outside the control limits is equivalent to rejecting the hypothesis of statistical control.

1.3.0 SPC in Discrete Parts Manufacturing :

The data obtained from the manufacturing process is to be plotted sequentially in time on a chart containing the target and the upper and lower control limits. This chart essentially provides a hypothesis test that the process mean is equal to the target against the alternative that it is not. The hypothesis test is based on an assumed in control model which dictates that the test statistic is uncorrelated . By comparing observed data with the in control model the hypothesis that the process is in control can either be accepted or rejected. If the limits are placed at 3σ the probability of falsely detecting an out of control condition is small. However , with these limits, the probability of detecting a small deviation is small and so it is usually augments with runs tests. To remain in a state of statistical control the variance should remain stable.

The key assumption behind these techniques is that the data obtained from a discrete parts manufacturing process is that the data are independent. This assumption is valid in discrete parts manufacturing . An example would be a part being machined on a lathe. The final dimension of a being machined now is not dependent on the diameter of the previously machined workpiece. The dimension may be affected by other factors such as a broken tool or operator error but not the previous workpiece.

Another major assumption behind the use of these control charts is the normality assumption . The central limit theorem can be used to justify the normality assumption when $\bar{x}$ is used. Since in discrete parts manufacturing the data is usually independent , the normality assumption holds good as long as the sample sizes are fairly large. Since the assumptions of data independence and normality can often be satisfied the control charts can be very effectively used as a process control technique in the discrete parts manufacturing industry.

1.4.0 SPC in Continous Process Manufacturing:

The assumption of data independence is vital for the application of traditional control charts . However, in a continous process environment this assumption is rarely satisfied. Often chemical processes have substantial inertia caused by process dead time . Hence the output of such a process is correlated in time. This results in the independence assumption not being satisfied even approximately. The products are often highly correlated from batch to batch .

The control charts will frequently give misleading results if the data are correlated. The control chart limits used to detect a significant deviation will be much too narrow, leading to many false alarms when in fact no shift in mean has occured or the expected number of observations before a change is falsely indicated will be very small. The effect of correlation on the performance of the control charts will be significant even in the presence of small correlation. So it is obvious that the existing statistical process control techniques cannot be used to control continous manufacturing processes. Traditional SPC takes the view that correlated processes are , by definition, out of control. When purely random disturbances are introduced to a continous process, correlated output will result. Since the true definition of an in control process is a "process driven purely by random noise " new approaches based on appropriate models are needed.

1.5.0 New Approach:

The new approach uses the existing statistical process control techniques but handles the data differently. This approach models the correlative structure as a time series model and uses that model to remove the autocorrelation from the data and plot the residuals on a control chart. Once the nature and parameters of the time series are known , the data to be plotted on a control chart is filtered accordingly. The proposed approaches for monitoring and controlling a correlated process is discussed in the subsequent chapters..

2.0 BACKGROUND:

It is appropriate at this stage to discuss the existing statistical process control techniques and related research work in this area before delving into our approach. The most commonly used control charts are introduced in the following section.

2.1.0 CONTROL CHARTS:

2.1.1 Shewart Control Chart:

Shewart Control Charts are named after their originator Walter A. Shewart who first laid out the basic philosophy behind the use of on-line control charts to monitor and control manufacturing processes. He suggested that data be plotted sequentially in time on a chart containing the target and the upper and lower control limits. The Shewart chart is still one of the most widely used control charts due to its simplicity. Based on the assumption of normality, this chart essentially provides a hypothesis test that the process mean or variance is equal to the target against the alternative that it is not. If the limits are placed at 3σ the probability of falsely detecting an out of control condition is small. However, the probability of detecting a small deviation is also small and so the standard Shewart procedure is usually augmented with run tests to increase its sensitivity.

To be in a state of statistical control the variance σ^2 should also remain stable. Therefore in discrete parts manufacturing industries, where these control charts are widely, used samples of n units are taken periodically and both the sample mean and the range or the sample standard deviation are plotted. The range or the sample standard deviation is an appropriate measure of the variability of the process. For example, five samples drawn from a process should give a reasonable measure of variability over the time period during which it was drawn. On the other hand five samples taken sequentially from a process might reflect the inherent process variability and show little about changing process variability , particularly if the process is autocorrelated. These charts work well under assumptions of normality and are fairly simple to implement. They are very useful for detecting large shifts in process mean or variance but not so effective in detecting small process shifts.

2.1.2. CUSUM Chart:

A major disadvantage of any Shewart control chart is that it only uses the information about the process contained in the most recent plotted point, and ignores any information given by the entire sequence of samples. So other criteria have been added to Shewart charts such as test for runs , the use of warning limits etc.,which attempt to incorporate information from the entire set of points into the decision procedure. These additional criteria however reduce the simplicity and ease of interpretation of the Shewart chart.

The cumulative sum or cusum control chart has been proposed [Montgomery] as an alternative to the Shewart control chart. It directly incorporates all of the information in the sequence of sample values by plotting the cumulative sums of the deviations of the sample values from a target value. For example, suppose that samples of size $n \geq 1$ are collected, and $\overline{x}_i$ is the average of the ith sample. Then if μ_o is the target for the process mean, the cumulative sum control chart is formed by plotting the quantity

$$S_m = \sum_{i=1}^{m} \overline{x}_i - \mu_o$$

against the sample number m. S_m is called the cumulative sum up to and including the mth sample. Because they combine information from several samples, cumulative - sum charts are more effective than Shewart charts for detecting small process shifts. If the process remains in control at the target value μ_o, the cumulative sum defined by the equation above should vary randomly about zero. However, if the mean shifts upwards to some value then an upward or a positive trend will develop in the cumulative sum. Conversely, if the mean shifts downwards to some value, then a downward or negative trend will develop . Therefore, if a trend develops in the plotted points either upward or downward, it should be considered as evidence that the process mean has shifted , and a search for some assignable cause should be performed. A V-mask is used a formal decision procedure for determining whether a process is in control or not.

Cumulative-sum control charts have several advantages over Shewart control chart in that they are far more effective in detecting relatively small shifts in the process mean . For small shifts they detect twice as quickly as a Shewart control chart. Also, the process shift is often easy to detect visually by the change of the slope in the plotted points. The cumulative-sum charts have some disadvantages. They are very slow to detect large process shifts . Diagnosis of patterns on the cumulative-sum control chart is very difficult because the usual assumption in pattern recognition is that the sequence of points is uncorrelated. The cumulative sums are not uncorrelated because successive values differ by only one observation . Consequently, the cumulative-sum control chart will often exhibit runs or other patterns that are an artifact of this correlation.

2.1.3 Geometric Moving Average Chart :

Control charts based on weighted averages are also very effective in detecting small process shifts. This control chart based on the geometric moving average or GMA is represented by

$$Z_t = \lambda \, \bar{x}_t + (1 - \lambda) \, Z_{t-1}$$

where $0 < \lambda < 1$ is a constant and the starting value required with the first sample at t=1 is

$$Z_0 = \bar{\bar{x}}$$

where $\bar{\bar{x}}$ is computed from historical data.

The weights given to samples decrease geometrically with the age of the sample mean. The GMA parameter λ determines how fast one discounts past data. When the parameter λ tends to unity only the current point is weighted and the GMA is equivalent to a Shewart chart. As λ tends towards zero., the GMA weights all points equally and resembles a CUSUM plot. Smaller values of λ should be used if early recognition of smaller shifts is desired and larger values for detecting large process shifts. The geometric average chart is an optimal control procedure for a process in which the mean in period t is related to the mean in period t - 1 .

2.2.0 Effect of Correlation on Control Charts:

The effect of serial correlation on the control charting procedures just described have been studied by researchers in the past and efforts have been taken to modify the control charts such that the presence of correlation does not seriously hamper the performance of the control charts.

T.J. Harris [1988] in his paper discusses the effects of correlation in sufficient detail and shows that they are substantial even in the presence of very small serial correlation. He also shows that sequential likelihood ratio developed to specifically account for serial correlation lead to modified CUSUM - like charts , but as the serial correlation becomes large, the probability of detecting a real shift in the mean becomes low. This results because the procedures are all based on testing a null hypothesis that the true mean of the process is on target. As serial correlation becomes large, it becomes more and more difficult to distinguish a shift in the mean from a temporary excursion in correlation.

Johnson and Bagshaw [1974] have studied the effect of serial correlation on the performance of CUSUM charts. They employ the theory of weak convergence of the cumulative sums to a Wiener process. They have provided a theoretical basis for studying the effects of serial correlation on the performance of the one - sided cusum test proposed by Page [1955]. Particular attention is placed on the first order autoregressive model and the first order moving average model. They not only studied the effect of correlation on the run length but also on the run length distribution . One primary conclusion of their work was that the cusum test is not very robust with respect to departures from independence.

In an another paper by Johnson and Bagshaw [1975] they have extended their previous work on the study of the effect of serial correlation on the performance of cusum tests by developing another approximation to the cumulative sums which allows one to study the run length distribution after a change in level has occured. Here again a first order autoregressive model and a first order moving average model has been considered. In this paper, they employ a somewhat different approximation , namely that of a Wiener process with drift, constrained to lie between a reflecting barrier and an absorbing barrier, they obtain results for the run length distribution after a change in the mean has occured. The new approximation was also used to study the run length distributions for in control situations. Their major emphasis was on studying run length distributions.

Vasilopoulos and Stamboulis [1978] in their paper discuss the modification of control chart limits in the presence of data correlation. When conditions of data independency no longer hold good, appropriate control limits for $\bar{x}$ and s charts have to be found since the introduction of serial correlation seems to greatly affect the distribution of $\bar{x}$ and s^2 . The ratio $V(\bar{x})$ indep / $V(\bar{x}$ AR(1)) is, for large n , proportional to $(1-\alpha)/(1+\alpha)$ while the ratio $V(s^2)$ indep / $V(s^2)$ AR (1) is proportional to $(1 - \alpha^2)/(1 + \alpha^2)$ where α is the correlation co-efficient. This can be interpreted to mean that the number of observations required , for the same variability , is modified by the factor $(1-\alpha)/(1+\alpha)$ in the $\bar{x}$ distribution and $(1 - \alpha^2)/(1 + \alpha^2)$ in the s^2 distribution. They discuss ways to analytically tabulate the quality control factors A, A_1, B_1, B_2 , B_3, B_4 in the presence of serial correlation . They extend this methodology for the AR(2) process also. Curves of the modified auxillary quality control factors have also been developed and presented in this paper.

3.0 PROPOSED APPROACH:

In the presence of serial correlation it is not possible to use control charts directly for reasons that were explained in the previous section. So it is necessary to filter the data obtained from the manufacturing process. The proposed approach is to directly model the correlative structure as a time series model, use that model to remove the auto-correlation from the data and apply control charts to the residuals. The uncorrelated residuals , which are then normally and independently distributed are plotted on a standard control chart with 3 σ limits. The direct observations from the process which are correlated cannot be plotted using the same limits . Instead their limits are so adjusted in such a way that the Type - I or alpha error for charts using both the uncorrelated residuals and the correlated output from the process are the same. Two time series models were considered for study here. One was the first order autoregressive model and the other was the first order moving average model.

For example, suppose that we could model the quality characteristic y(t) as

$$y(t) = \phi * y(t-1) + z(t) \text{ ...... } 1$$

where ϕ is a constant and z(t) is a sequence of uncorrelated NID$(0,\sigma^2)$ shocks . This would be representative of a first order autoregressive model. This equation is composed of two parts, the prediction term [ϕ* y(t-1)]and the error term [z(t)]. When the process output is observed the above equation can then be re-written to determine the z(t).

$$Z(t) = Y(t) - \phi * Y(t-1) \ldots\ldots 2$$

Since the z(t) is nothing but a sequence of uncorrelated shocks they are normally and independently distributed and may be plotted on the control charts using 3σ limits. The correlation coefficient ϕ is a constant and is assumed known . Since the values of the observations at time t-1 and t are measured, z(t) can be determined from equation 2. This data can then be plotted on a control chart to study and control the process. Since the z(t) are uncorrelated they are independent and normal , and 3σ limits can be used to determine the upper and lower control limits.

For comparison purposes, we also investigate the usual approach; a control chart on the process output. However, the observations or the process output cannot be plotted using the same control limits [Stamboulis & Vasilopoulus, 1974] because the control chart limits used to detect a significant deviation will be much too narrow leading to many false alarms when in fact no shift in mean has occured or the expected number of observations before a change is falsely indicated will be very small. The effect of correlation on the performance of control charts will be significant even in the presence of small correlation. Rather than setting the control limits at 3σ , the control chart limits are set at $k\sigma$ such that the probability of generating a false alarm will be the same as that of a chart based on normally and independently distributed data with 3σ limits . The value of k was arrived at by simulation using a trial and error approach. They were adjusted in such a way that the alpha error for the charts were the same and equal to 0.0027.Modification of the control chart limits such that the alpha errors for both charts are the same provides a common basis for comparison of the performance of the two charts.

Another example would be to model the quality characteristic Y (t) as

$$Y(t) = Z(t) - \theta * Z(t-1) \text{ ...... } 3$$

where θ is a constant and z(t) is a sequence of uncorrelated NID shocks. This would be representative of a first order moving average model. Equation 3 can be re-written to yield equation 4

$$z(t) = y(t) + \theta * z(t-1) \text{ ...... } 4$$

The same approach prescribed for a first order autoregressive model can be adopted for a first order moving average model. The residuals or shocks and the observations are plotted on two different charts with different control limits and their performances compared.

3.1.0 Types of Shifts:

An out of control condition of the process often results from a shift in the mean of the process. It could be either an upward or a downward shift in the level of the process. The aim of statistical process control is to detect as quickly as possible any change in the level of the mean from the target value. A shift in the level of the process can occur due to various reasons.

ARIMA models are generated by passing white noise (NID(0,σ^2 random shocks) through a process model to generate correlated processes. This model agrees closely with our view of the behaviour of many real world continous processes. That is , when in control, inputs can be modelled by white noise.

It is possible to envision two extreme cases as to how an assignable cause could affect the process. A change in the level of an input to the process could be modelled as a step change in the mean of the z(t) driving the model. It is also possible to envision an assignable cause at the output resulting in a step change in y(t). The black box approach taken by ARIMA models to represent the process is simplistic for our purposes in that a level change may occur at some point within the process. However we take the view that an SPC procedure that effectively detects shifts at both the input and the output should be effective in general. While others have concentrated on detecting shifts at the output we feel that it is important to consider both cases.

For example, consider a first order autoregressive model

$$Y(t) = \phi * Y(t-1) + z(t)$$

where ϕ is a constant and z(t) is a sequence of uncorrelated shocks.

The input term usually represents the input variability of the process when the process is in control. No matter how well designed a process may be there is bound to be some inherent variability which cannot be controlled. This variability is the cumulative effect of many small and essentially uncontrollable causes. In the time series model, these random shocks are modelled as inputs to the process. For example if the shift in the mean of the process was due to this reason then

$$z'(t) = z(t) + \mu$$

where μ is a constant and represents a shift. If the distribution of z(t) before the shift had occured was NID(0,1) and if the magnitude of the shift was of the order of 1σ then the distribution of the z(t) would be NID(1,1). Now the values of the observations would change to

$$y'(t) = \phi * y'(t-1) + z'(t)$$

or

$$y'(t) = \phi * y'(t-1) + z(t) + \mu$$

The prediction terms' behaviour can be predicted because all the interrelationships which make up this behaviour are known . For example the shift in the prediction term can be represented as

$$\phi * y'(t-1) = \phi * y(t-1) + \mu)$$

So

$$y'(t) = \phi * y'(t-1) + z'(t)$$

or

$$y'(t) = \phi * y(t-1) + \mu + z'(t)$$

Therefore $\quad z'(t) = y'(t) - \phi * y'(t-1)$

So a shift in the level of the process output [y(t)] can either be due to a change in the level of the input or output. A change in the level of the process due to the increase in mean of the error or input term leads to an increase in the level of the process output. This can be due to the fact that the input variability of the process has increased due to flow control problems.

On the other hand is the change in the level of the process due to a change in the level of the prediction term . The prediction term can be accurately established because all the interrelationships between the inputs and other factors which influence the process output are clearly known. Therefore a change in the level of any one factor vis-a-vis another factor leads to a shift in the mean of the process.

The mean of the process could also shift upwards. If the mean of the process did actually shift its performance in so far as its ability to detect the shift was also studied. This was done by introducing a shift in the process . The performance of the charts were measured by the Type - I error or alpha - error and average run length. The control limits of the two charts were adjusted in such a manner that Type - I error was equal for both charts. So , for an equal alpha-error, the performance of the charts were compared by their average run lengths. The objective here was to see which of the two charts was able to detect a shift in the mean of the process faster . This meant having shorter run lengths when there was a shift in the mean of the process and having equivalent run lengths when there was no shift in the mean of the manufacturing process . The same approach was adopted for the first order moving average model.

3.2.0 Simulation Model :

A simulation model was developed using Fortran. The main reason for simulation was because of the inability to determine the average run lengths analytically. The two processes under consideration , the first order autoregressive or AR(1) model and the first order moving average or MA(1) model were generated using a pseudo random number generator and then converting them to a normal distribution with mean 0 and standard deviation 1. This normal (0,1) generator was then used to generate a time series model. For example if the model under consideration was AR(1) then

$$y(0) = 0$$

$$y(1) = \phi * y(0) + \text{Normal } (0,1)$$

$$y(2) = \phi * y(1) + \text{Normal } (0,1)$$

$$\vdots$$

$$y(n) = \phi * y(n-1) + \text{Normal } (0,1)$$

where ϕ is a constant and $-1 \leq \phi \leq 1$.

After the generation of a time series the observations y(t), the uncorrelated shocks are filtered. The uncorrelated shocks are nothing but the normal (0,1) distribution used to generate the time series in the first place. Therefore

$$z(t) = y(t) - \phi * y(t-1)$$

The residuals z(t) are then plotted on a Geometric Moving Average (GMA) chart using 3σ limits. For the data to be plotted on a GMA chart the data is first transformed to its geometric moving average.

$$GMA[z(t)] = \lambda * z(t) + (1-\lambda)\, GMA[z(t-1)]$$

where λ is a constant and $0 < \lambda \leq 1$. The parameter λ determines how fast one discounts past data. The reason for choosing a GMA chart over a Shewart control chart is because of its ability to detect small shifts in mean faster. The upper and lower control limits for a

GMA chart are determined as follows.

$$UCL = \bar{z}(t) + 3\sigma\sqrt{\frac{\lambda}{n(2-\lambda)}}$$

$$LCL = \bar{z}(t) - 3\sigma\sqrt{\frac{\lambda}{n(2-\lambda)}}$$

where n =1 is the sample size and hence $z(t) = \bar{z}(t)$. Since the z(t) is normally and independently distributed [0,1] , using the 3σ limits means the probability of falsely indicating an out of control condition is equal to 0.0027 or .27% of the time. In other words the alpha error is equal to 0.0027.

However, the y(t) are correlated and they cannot be plotted using 3σ limits because of the violation of the independence assumption leading to the control limits being either too narrow or too broad. This leads to many false alarms or slow detection of shifts. So it is proposed to set the control limits at $k\sigma$ such that the alpha error at this value of $k\sigma$ was equal to 0.0027. The obseravtions were plotted on a GMA chart using these limits.

$$GMA[y(t)] = \lambda * y(t) + (1-\lambda)\, y(t-1)$$

$$UCL = y(t) + k\sigma_y\sqrt{\frac{\lambda}{n(2-\lambda)}}$$

$$LCL = y(t) - k\sigma_y\sqrt{\frac{\lambda}{n(2-\lambda)}}$$

The in-control alpha was determined before the occurence of a shift . A process whose mean has not shifted is deemed in a state of control and

$$\alpha = \text{\# of observations outside limts / Total \# of observations}$$

So by changing the control limits the value of alpha could either be increased or decreased. The control limits were set so as to yield an alpha value of 0.0027. The alpha was based on a total of 50000 observations.

The next step was to introduce a shift in the mean of the process to check the performance of the chart in detecting shifts. As explained in the previous section, shifts in the mean could either be due to the error term or due to the prediction term. Upon introducing a shift to the process the observations and the uncorrelated shocks are plotted on a GMA chart. If a point plotted outside the control limits the process was deemed to be out of control. Once an observation was found to be out of control its GMA was reset to zero meaning that the process was brought back to an in control state. The same process was then repeated. The number of observations between two out of control points was taken as the run length. This was carried on repetitively until 1000 out of control observations were detected. As stated earlier, each time a point plotted outside the control limits the GMA of that observation was reset to zero. Further observations were plotted and the entire process was repeated. The values for run lengths were averaged to obtain the average run length.

3.3.0.Factors :

Several factors were considered for the purposes of simulation to study this approach under different environments.

3.3.1.Time Series Models:

Two time series models were considered. They were the first order autoregressive or AR(1) model and the first order moving average or MA(1) model.

3.3.2.ARIMA Model Parameters:

Two time series models namely the AR(1) model and the MA(1) model were used. Each model was tested at different levels of the model coefficient.This factor had nine levels from 0.9, 0.7,0.5,0.3,....-0.5,-0.7.

3.3.3.Geometric Moving Average Parameter:

The geometric moving average parameter λ determines how fast one discounts past data. A value of 1 would reduce the GMA chart to a Shewart control chart . Three levels were tried. They were 0.25, 0.50 and 0.75.

3.3.4.Shifts :

The level of shift in the mean of the process was another important factor. This factor had six levels. They were 0σ, 0.5σ,1σ, 2σ, 3σ and 4σ. The values of σ_Y were determined using simulation.

3.4.0 Performance Measures :

The most important performance measure is the average run length . Run length is the number of points or observations plotted between two consecutive out of control points. Several run lengths are averaged to obtain the average run length. The average run lengths for charts plotted with the observations [y(t)] and for charts plotted with the uncorrelated shocks [z(t)] are compared to test the performance of the proposed approach of using the uncorrelated shocks to detect the shift in mean of the process rather than using the correlated observations. The two charts are compared on an equal footing because the control limits are set such that the alpha levels for both charts are the same. Therefore for equal

alpha the chart which detects a shift in the mean faster is obviously better. In other words the chart which has a shorter run length for a shift in the mean of the process and longer run length when there is no shift is more desirable.

3.5.0 Environments :

The two time series models namely the first order auto regressive and moving average models were simulated extensively under different environments.The different environments were created by changing the levels of different factors. For example for a geometric moving average parameter of $\lambda = 0.25$, nine levels of correlation co-efficient from -0.7 to + 0.9 and six levels of the shift in the mean from 0σ to 4σ means a total of 54 runs. The same experiments were carried out for different values of λ (0.25, 0.50, 0.70). The whole set of 162 runs (54 x 3) were repeated for the two types of shifts (Error or Prediction term) and for each time series model (AR(1) & MA(1)).

3.6.0. Results :

The results are shown in Tables 1 - 4. These tables show the control limit factors used for plotting GMA[y's], correlation co-efficient , the mean and standard deviation of the correlated observations and the uncorrelated shocks both before and after the shift , the in control alpha for charts plotting both the observations and the residuals , the average run lengths and the magnitude of the shift in the process. The results are discussed in the following sections.

TABLE 1

MA(1) Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.97***********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	1.585	0.00	0.00	1.35	1.00	0.0	370.37	370.37	0.00	1.35	0.00	1.00
0.9	1.585	0.00	0.00	1.35	1.00	0.5	301.20	46.31	0.05	1.35	0.50	1.00
0.9	1.585	0.00	0.00	1.35	1.00	1.0	233.64	10.75	0.10	1.35	1.00	1.01
0.9	1.585	0.00	0.00	1.35	1.00	2.0	90.91	3.57	0.20	1.35	2.00	1.02
0.9	1.585	0.00	0.00	1.35	1.00	3.0	42.51	2.22	0.30	1.35	3.01	1.02
0.9	1.585	0.00	0.00	1.35	1.00	4.0	23.13	1.72	0.40	1.35	4.02	1.03
0.7	1.680	0.00	0.00	1.22	1.00	0.0	370.37	370.37	0.00	1.22	0.00	1.00
0.7	1.680	0.00	0.00	1.22	1.00	0.5	140.45	46.31	0.15	1.22	0.50	1.00
0.7	1.680	0.00	0.00	1.22	1.00	1.0	41.12	10.75	0.30	1.22	1.00	1.01
0.7	1.680	0.00	0.00	1.22	1.00	2.0	10.80	3.57	0.60	1.22	2.00	1.02
0.7	1.680	0.00	0.00	1.22	1.00	3.0	5.96	2.22	0.90	1.24	3.01	1.02
0.7	1.680	0.00	0.00	1.22	1.00	4.0	4.20	1.72	1.20	1.25	4.02	1.03
0.5	1.910	0.00	0.00	1.12	1.00	0.0	364.96	370.37	0.00	1.12	0.00	1.00
0.5	1.910	0.00	0.00	1.12	1.00	0.5	68.31	46.31	0.25	1.12	0.50	1.00
0.5	1.910	0.00	0.00	1.12	1.00	1.0	17.49	10.75	0.50	1.12	1.00	1.01
0.5	1.910	0.00	0.00	1.12	1.00	2.0	5.66	3.57	1.00	1.14	2.00	1.02
0.5	1.910	0.00	0.00	1.12	1.00	3.0	3.42	2.22	1.50	1.15	3.01	1.02
0.5	1.910	0.00	0.00	1.12	1.00	4.0	2.49	1.72	2.00	1.16	4.02	1.03
0.3	2.290	0.00	0.00	1.04	1.00	0.0	373.13	370.37	0.00	1.04	0.00	1.00
0.3	2.290	0.00	0.00	1.04	1.00	0.5	51.23	46.31	0.35	1.04	0.50	1.00
0.3	2.290	0.00	0.00	1.04	1.00	1.0	12.74	10.75	0.70	1.05	1.00	1.01
0.3	2.290	0.00	0.00	1.04	1.00	2.0	4.30	3.57	1.40	1.07	2.00	1.02
0.3	2.290	0.00	0.00	1.04	1.00	3.0	2.62	2.22	2.10	1.08	3.01	1.02
0.3	2.290	0.00	0.00	1.04	1.00	4.0	1.96	1.72	2.82	1.08	4.02	1.03
0.1	2.725	0.00	0.00	1.01	1.00	0.0	367.65	370.37	0.00	1.01	0.00	1.00
0.1	2.725	0.00	0.00	1.01	1.00	0.5	45.98	46.31	0.45	1.01	0.50	1.00
0.1	2.725	0.00	0.00	1.01	1.00	1.0	10.93	10.75	0.90	1.01	1.00	1.01
0.1	2.725	0.00	0.00	1.01	1.00	2.0	3.71	3.57	1.80	1.03	2.00	1.02
0.1	2.725	0.00	0.00	1.01	1.00	3.0	2.31	2.22	2.70	1.03	3.01	1.02
0.1	2.725	0.00	0.00	1.01	1.00	4.0	1.76	1.72	3.62	1.04	4.02	1.03

TABLE 1

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	3.160	0.00	0.00	1.01	1.00	0.0	373.13	370.37	0.00	1.01	0.00	1.00
-0.1	3.160	0.00	0.00	1.01	1.00	0.5	44.09	46.31	0.55	1.01	0.50	1.00
-0.1	3.160	0.00	0.00	1.01	1.00	1.0	10.30	10.75	1.10	1.01	1.00	1.01
-0.1	3.160	0.00	0.00	1.01	1.00	2.0	3.42	3.57	2.19	1.03	2.00	1.02
-0.1	3.160	0.00	0.00	1.01	1.00	3.0	2.16	2.22	3.31	1.03	3.01	1.02
-0.1	3.160	0.00	0.00	1.01	1.00	4.0	1.66	1.72	4.42	1.03	4.02	1.03
-0.3	3.460	0.00	0.00	1.05	1.00	0.0	367.65	370.37	0.00	1.05	0.00	1.00
-0.3	3.460	0.00	0.00	1.05	1.00	0.5	42.20	46.31	0.65	1.05	0.50	1.00
-0.3	3.460	0.00	0.00	1.05	1.00	1.0	9.78	10.75	1.31	1.05	1.00	1.01
-0.3	3.460	0.00	0.00	1.05	1.00	2.0	3.30	3.57	2.58	1.06	2.00	1.02
-0.3	3.460	0.00	0.00	1.05	1.00	3.0	2.08	2.22	3.92	1.05	3.01	1.02
-0.3	3.460	0.00	0.00	1.05	1.00	4.0	1.57	1.72	5.22	1.06	4.02	1.03
-0.5	3.650	0.00	0.00	1.12	1.00	0.0	370.37	370.37	0.00	1.12	0.00	1.00
-0.5	3.650	0.00	0.00	1.12	1.00	0.5	40.87	46.31	0.75	1.12	0.50	1.00
-0.5	3.650	0.00	0.00	1.12	1.00	1.0	9.50	10.75	1.51	1.13	1.00	1.01
-0.5	3.650	0.00	0.00	1.12	1.00	2.0	3.20	3.57	2.98	1.14	2.00	1.02
-0.5	3.650	0.00	0.00	1.12	1.00	3.0	2.04	2.22	4.53	1.13	3.01	1.02
-0.5	3.650	0.00	0.00	1.12	1.00	4.0	1.50	1.72	6.02	1.12	4.02	1.03
-0.7	3.780	0.00	0.00	1.22	1.00	0.0	373.13	370.37	0.00	1.22	0.00	1.00
-0.7	3.780	0.00	0.00	1.22	1.00	0.5	41.52	46.31	0.85	1.22	0.50	1.00
-0.7	3.780	0.00	0.00	1.22	1.00	1.0	9.64	10.75	1.71	1.23	1.00	1.01
-0.7	3.780	0.00	0.00	1.22	1.00	2.0	3.18	3.57	3.38	1.24	2.00	1.02
-0.7	3.780	0.00	0.00	1.22	1.00	3.0	2.03	2.22	5.13	1.22	3.01	1.02
-0.7	3.780	0.00	0.00	1.22	1.00	4.0	1.49	1.72	6.82	1.22	4.02	1.03

TABLE 2

MA(1) Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.97***********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	1.585	0.00	0.00	1.35	1.00	0.0	370.37	370.37	0.00	1.35	0.00	1.00
0.9	1.585	0.00	0.00	1.35	1.00	0.5	15.08	1.32	0.50	1.35	5.01	1.02
0.9	1.585	0.00	0.00	1.35	1.00	1.0	5.40	1.00	1.00	1.37	9.98	1.01
0.9	1.585	0.00	0.00	1.35	1.00	2.0	2.52	1.00	2.00	1.40	19.99	1.01
0.9	1.585	0.00	0.00	1.35	1.00	3.0	1.72	1.00	3.00	1.40	30.00	1.01
0.9	1.585	0.00	0.00	1.35	1.00	4.0	1.35	1.00	4.00	1.38	40.01	1.01
0.7	1.680	0.00	0.00	1.22	1.00	0.0	370.37	370.37	0.00	1.22	0.00	1.00
0.7	1.680	0.00	0.00	1.22	1.00	0.5	14.61	4.61	0.50	1.22	1.67	1.01
0.7	1.680	0.00	0.00	1.22	1.00	1.0	5.25	2.03	1.00	1.25	3.35	1.02
0.7	1.680	0.00	0.00	1.22	1.00	2.0	2.40	1.02	2.00	1.27	6.65	1.02
0.7	1.680	0.00	0.00	1.22	1.00	3.0	1.64	1.00	3.01	1.27	9.98	1.01
0.7	1.680	0.00	0.00	1.22	1.00	4.0	1.29	1.00	4.00	1.25	13.32	1.01
0.5	1.910	0.00	0.00	1.12	1.00	0.0	364.96	370.37	0.00	1.12	0.00	1.00
0.5	1.910	0.00	0.00	1.12	1.00	0.5	17.49	10.77	0.50	1.12	1.00	1.01
0.5	1.910	0.00	0.00	1.12	1.00	1.0	5.66	3.57	1.00	1.14	2.00	1.02
0.5	1.910	0.00	0.00	1.12	1.00	2.0	2.49	1.72	2.00	1.16	4.02	1.03
0.5	1.910	0.00	0.00	1.12	1.00	3.0	1.70	1.08	3.01	1.16	5.99	1.02
0.5	1.910	0.00	0.00	1.12	1.00	4.0	1.29	1.00	4.00	1.14	7.98	1.01
0.3	2.290	0.00	0.00	1.04	1.00	0.0	373.13	370.37	0.00	1.04	0.00	1.00
0.3	2.290	0.00	0.00	1.04	1.00	0.5	24.46	21.34	0.50	1.05	0.71	1.01
0.3	2.290	0.00	0.00	1.04	1.00	1.0	6.77	5.79	1.01	1.06	1.43	1.02
0.3	2.290	0.00	0.00	1.04	1.00	2.0	2.78	2.35	2.00	1.08	2.87	1.03
0.3	2.290	0.00	0.00	1.04	1.00	3.0	1.85	1.59	3.02	1.07	4.30	1.03
0.3	2.290	0.00	0.00	1.04	1.00	4.0	1.40	1.12	4.01	1.06	5.70	1.02
0.1	2.725	0.00	0.00	1.01	1.00	0.0	367.65	370.37	0.00	1.01	0.00	1.00
0.1	2.725	0.00	0.00	1.01	1.00	0.5	36.58	36.75	0.50	1.01	0.55	1.01
0.1	2.725	0.00	0.00	1.01	1.00	1.0	9.01	8.78	1.01	1.02	1.12	1.01
0.1	2.725	0.00	0.00	1.01	1.00	2.0	3.23	3.13	1.99	1.03	2.21	1.02
0.1	2.725	0.00	0.00	1.01	1.00	3.0	2.07	2.03	3.02	1.03	3.35	1.02
0.1	2.725	0.00	0.00	1.01	1.00	4.0	1.58	1.52	4.01	1.03	4.46	1.01

TABLE 2

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	3.160	0.00	0.00	1.01	1.00	0.0	373.13	370.37	0.00	1.01	0.00	1.00
-0.1	3.160	0.00	0.00	1.01	1.00	0.5	53.71	55.68	0.50	1.01	0.45	1.00
-0.1	3.160	0.00	0.00	1.01	1.00	1.0	12.31	12.81	1.00	1.01	0.91	1.01
-0.1	3.160	0.00	0.00	1.01	1.00	2.0	3.96	4.14	2.00	1.03	1.81	1.02
-0.1	3.160	0.00	0.00	1.01	1.00	3.0	2.38	2.48	3.01	1.03	2.73	1.03
-0.1	3.160	0.00	0.00	1.01	1.00	4.0	1.82	1.87	4.03	1.03	3.67	1.02
-0.3	3.460	0.00	0.00	1.05	1.00	0.0	367.65	370.37	0.00	1.05	0.00	1.00
-0.3	3.460	0.00	0.00	1.05	1.00	0.5	70.62	75.53	0.50	1.05	0.38	1.00
-0.3	3.460	0.00	0.00	1.05	1.00	1.0	16.67	18.29	1.00	1.05	0.76	1.01
-0.3	3.460	0.00	0.00	1.05	1.00	2.0	4.73	5.14	2.00	1.06	1.54	1.02
-0.3	3.460	0.00	0.00	1.05	1.00	3.0	2.73	2.98	3.00	1.07	2.30	1.03
-0.3	3.460	0.00	0.00	1.05	1.00	4.0	2.04	2.17	4.02	1.05	3.09	1.02
-0.5	3.650	0.00	0.00	1.12	1.00	0.0	370.37	370.37	0.00	1.12	0.00	1.00
-0.5	3.650	0.00	0.00	1.12	1.00	0.5	87.87	97.66	0.50	1.12	0.33	1.00
-0.5	3.650	0.00	0.00	1.12	1.00	1.0	22.71	24.98	0.99	1.13	0.66	1.01
-0.5	3.650	0.00	0.00	1.12	1.00	2.0	5.69	6.38	2.01	1.13	1.34	1.01
-0.5	3.650	0.00	0.00	1.12	1.00	3.0	3.21	3.57	2.98	1.14	2.00	1.02
-0.5	3.650	0.00	0.00	1.12	1.00	4.0	2.30	2.54	4.01	1.13	2.67	1.03
-0.7	3.780	0.00	0.00	1.22	1.00	0.0	373.13	370.37	0.00	1.22	0.00	1.00
-0.7	3.780	0.00	0.00	1.22	1.00	0.5	111.86	116.55	0.50	1.22	0.29	1.00
-0.7	3.780	0.00	0.00	1.22	1.00	1.0	29.51	32.74	1.00	1.23	0.59	1.01
-0.7	3.780	0.00	0.00	1.22	1.00	2.0	6.98	7.80	2.02	1.24	1.19	1.02
-0.7	3.780	0.00	0.00	1.22	1.00	3.0	3.74	4.26	3.00	1.24	1.76	1.02
-0.7	3.780	0.00	0.00	1.22	1.00	4.0	2.60	2.93	4.01	1.24	2.35	1.03

TABLE 3

AR(1) Model
*********** CONTROL LIMIT FACTOR FOR Zs = 2.97***********
***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT
ALPHAY=ALPHAZ=0.0027
LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	5.720	-.01	0.00	2.33	1.00	0.0	362.32	370.37	-0.01	2.33	0.00	1.00
0.9	5.720	-.01	0.00	2.33	1.00	0.5	10.66	46.31	5.01	2.37	0.50	1.00
0.9	5.720	-.01	0.00	2.33	1.00	1.0	3.02	10.75	9.89	2.27	1.00	1.01
0.9	5.720	-.01	0.00	2.33	1.00	2.0	1.41	3.57	20.06	2.19	2.00	1.02
0.9	5.720	-.01	0.00	2.33	1.00	3.0	1.00	2.22	29.76	2.12	3.01	1.02
0.9	5.720	-.01	0.00	2.33	1.00	4.0	1.00	1.72	39.77	2.12	4.02	1.03
0.7	4.880	0.00	0.00	1.41	1.00	0.0	367.65	370.37	0.00	1.41	0.00	1.00
0.7	4.880	0.00	0.00	1.41	1.00	0.5	26.92	46.31	1.66	1.43	0.50	1.00
0.7	4.880	0.00	0.00	1.41	1.00	1.0	6.01	10.75	3.35	1.42	1.00	1.01
0.7	4.880	0.00	0.00	1.41	1.00	2.0	2.26	3.57	6.69	1.39	2.00	1.02
0.7	4.880	0.00	0.00	1.41	1.00	3.0	1.55	2.22	10.04	1.39	3.01	1.02
0.7	4.880	0.00	0.00	1.41	1.00	4.0	1.01	1.72	13.25	1.36	4.02	1.03
0.5	4.180	0.00	0.00	1.16	1.00	0.0	367.65	370.37	0.00	1.16	0.00	1.00
0.5	4.180	0.00	0.00	1.16	1.00	0.5	35.28	46.31	1.00	1.17	0.50	1.00
0.5	4.180	0.00	0.00	1.16	1.00	1.0	7.87	10.75	2.03	1.18	1.00	1.01
0.5	4.180	0.00	0.00	1.16	1.00	2.0	2.72	3.57	4.00	1.18	2.00	1.02
0.5	4.180	0.00	0.00	1.16	1.00	3.0	1.86	2.22	6.05	1.17	3.01	1.02
0.5	4.180	0.00	0.00	1.16	1.00	4.0	1.24	1.72	8.01	1.16	4.02	1.03
0.3	3.630	0.00	0.00	1.05	1.00	0.0	373.13	370.37	0.00	1.05	0.00	1.00
0.3	3.630	0.00	0.00	1.05	1.00	0.5	40.37	46.31	0.71	1.05	0.50	1.00
0.3	3.630	0.00	0.00	1.05	1.00	1.0	9.15	10.75	1.44	1.06	1.00	1.01
0.3	3.630	0.00	0.00	1.05	1.00	2.0	3.11	3.57	2.84	1.07	2.00	1.02
0.3	3.630	0.00	0.00	1.05	1.00	3.0	2.01	2.22	4.31	1.06	3.01	1.02
0.3	3.630	0.00	0.00	1.05	1.00	4.0	1.48	1.72	5.73	1.05	4.02	1.03
0.1	3.170	0.00	0.00	1.01	1.00	0.0	370.37	370.37	0.00	1.01	0.00	1.00
0.1	3.170	0.00	0.00	1.01	1.00	0.5	43.70	46.31	0.55	1.01	0.50	1.00
0.1	3.170	0.00	0.00	1.01	1.00	1.0	10.20	10.75	1.11	1.01	1.00	1.01
0.1	3.170	0.00	0.00	1.01	1.00	2.0	3.39	3.57	2.21	1.03	2.00	1.02
0.1	3.170	0.00	0.00	1.01	1.00	3.0	2.14	2.22	3.35	1.03	3.01	1.02
0.1	3.170	0.00	0.00	1.01	1.00	4.0	1.65	1.72	4.46	1.03	4.02	1.03

TABLE 3

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	2.730	0.00	0.00	1.01	1.00	0.0	362.32	370.37	0.00	1.01	0.00	1.00
-0.1	2.730	0.00	0.00	1.01	1.00	0.5	44.78	46.31	0.45	1.01	0.50	1.00
-0.1	2.730	0.00	0.00	1.01	1.00	1.0	10.73	10.75	0.91	1.01	1.00	1.01
-0.1	2.730	0.00	0.00	1.01	1.00	2.0	3.66	3.57	1.82	1.03	2.00	1.02
-0.1	2.730	0.00	0.00	1.01	1.00	3.0	2.29	2.22	2.73	1.03	3.01	1.02
-0.1	2.730	0.00	0.00	1.01	1.00	4.0	1.75	1.72	3.66	1.04	4.02	1.03
-0.3	2.340	0.00	0.00	1.05	1.00	0.0	362.32	370.37	0.00	1.05	0.00	1.00
-0.3	2.340	0.00	0.00	1.05	1.00	0.5	45.79	46.31	0.38	1.05	0.50	1.00
-0.3	2.340	0.00	0.00	1.05	1.00	1.0	11.40	10.75	0.77	1.05	1.00	1.01
-0.3	2.340	0.00	0.00	1.05	1.00	2.0	3.94	3.57	1.54	1.08	2.00	1.02
-0.3	2.340	0.00	0.00	1.05	1.00	3.0	2.45	2.22	2.31	1.09	3.01	1.02
-0.3	2.340	0.00	0.00	1.05	1.00	4.0	1.85	1.72	3.10	1.08	4.02	1.03
-0.5	2.000	0.00	0.00	1.16	1.00	0.0	362.32	370.37	0.00	1.16	0.00	1.00
-0.5	2.000	0.00	0.00	1.16	1.00	0.5	49.93	46.31	0.33	1.16	0.50	1.00
-0.5	2.000	0.00	0.00	1.16	1.00	1.0	12.82	10.75	0.67	1.16	1.00	1.01
-0.5	2.000	0.00	0.00	1.16	1.00	2.0	4.31	3.57	1.33	1.19	2.00	1.02
-0.5	2.000	0.00	0.00	1.16	1.00	3.0	2.66	2.22	2.00	1.21	3.01	1.02
-0.5	2.000	0.00	0.00	1.16	1.00	4.0	1.99	1.72	2.68	1.21	4.02	1.03
-0.7	1.705	0.00	0.00	1.41	1.00	0.0	364.96	370.37	0.00	1.41	0.00	1.00
-0.7	1.705	0.00	0.00	1.41	1.00	0.5	66.84	46.31	0.29	1.41	0.50	1.00
-0.7	1.705	0.00	0.00	1.41	1.00	1.0	16.68	10.75	0.59	1.41	1.00	1.01
-0.7	1.705	0.00	0.00	1.41	1.00	2.0	5.22	3.57	1.18	1.44	2.00	1.02
-0.7	1.705	0.00	0.00	1.41	1.00	3.0	3.21	2.22	1.76	1.45	3.01	1.02
-0.7	1.705	0.00	0.00	1.41	1.00	4.0	2.31	1.72	2.36	1.47	4.02	1.03

TABLE 4

AR(1) Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.97**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	5.720	-.01	0.00	2.33	1.00	0.0	362.32	370.37	-0.01	2.33	0.00	1.00
0.9	5.720	-.01	0.00	2.33	1.00	0.5	294.12	396.83	0.49	2.33	0.05	1.00
0.9	5.720	-.01	0.00	2.33	1.00	1.0	202.43	342.47	0.99	2.33	0.10	1.00
0.9	5.720	-.01	0.00	2.33	1.00	2.0	77.64	183.15	1.99	2.33	0.20	1.00
0.9	5.720	-.01	0.00	2.33	1.00	3.0	34.71	110.38	2.98	2.34	0.30	1.00
0.9	5.720	-.01	0.00	2.33	1.00	4.0	17.87	70.22	3.96	2.38	0.40	1.00
0.7	4.880	0.00	0.00	1.41	1.00	0.0	367.65	370.37	0.00	1.41	0.00	1.00
0.7	4.880	0.00	0.00	1.41	1.00	0.5	212.77	261.78	0.50	1.41	0.15	1.00
0.7	4.880	0.00	0.00	1.41	1.00	1.0	78.37	110.38	1.00	1.41	0.30	1.00
0.7	4.880	0.00	0.00	1.41	1.00	2.0	18.04	30.81	1.99	1.43	0.60	1.01
0.7	4.880	0.00	0.00	1.41	1.00	3.0	7.36	13.02	3.04	1.43	0.90	1.01
0.7	4.880	0.00	0.00	1.41	1.00	4.0	4.39	7.56	4.00	1.41	1.21	1.02
0.5	4.180	0.00	0.00	1.16	1.00	0.0	367.65	370.37	0.00	1.16	0.00	1.00
0.5	4.180	0.00	0.00	1.16	1.00	0.5	124.69	139.66	0.50	1.16	0.25	1.00
0.5	4.180	0.00	0.00	1.16	1.00	1.0	35.28	46.31	1.00	1.17	0.50	1.00
0.5	4.180	0.00	0.00	1.16	1.00	2.0	7.87	10.75	2.03	1.18	1.00	1.01
0.5	4.180	0.00	0.00	1.16	1.00	3.0	3.98	5.30	2.99	1.17	1.51	1.02
0.5	4.180	0.00	0.00	1.16	1.00	4.0	2.72	3.57	4.00	1.18	2.00	1.02
0.3	3.630	0.00	0.00	1.05	1.00	0.0	373.13	370.37	0.00	1.05	0.00	1.00
0.3	3.630	0.00	0.00	1.05	1.00	0.5	79.49	90.25	0.50	1.05	0.35	1.00
0.3	3.630	0.00	0.00	1.05	1.00	1.0	19.47	22.72	0.99	1.06	0.69	1.01
0.3	3.630	0.00	0.00	1.05	1.00	2.0	5.09	5.93	2.00	1.06	1.40	1.02
0.3	3.630	0.00	0.00	1.05	1.00	3.0	2.94	3.36	2.99	1.07	2.09	1.02
0.3	3.630	0.00	0.00	1.05	1.00	4.0	2.13	2.39	4.02	1.06	2.81	1.03
0.1	3.170	0.00	0.00	1.01	1.00	0.0	370.37	370.37	0.00	1.01	0.00	1.00
0.1	3.170	0.00	0.00	1.01	1.00	0.5	54.11	56.82	0.50	1.01	0.45	1.00
0.1	3.170	0.00	0.00	1.01	1.00	1.0	12.35	13.02	1.00	1.01	0.90	1.01
0.1	3.170	0.00	0.00	1.01	1.00	2.0	4.00	4.19	1.99	1.02	1.79	1.02
0.1	3.170	0.00	0.00	1.01	1.00	3.0	2.39	2.51	3.01	1.03	2.70	1.03
0.1	3.170	0.00	0.00	1.01	1.00	4.0	1.83	1.88	4.03	1.03	3.63	1.03

TABLE 4

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	2.730	0.00	0.00	1.01	1.00	0.0	362.32	370.37	0.00	1.01	0.00	1.00
-0.1	2.730	0.00	0.00	1.01	1.00	0.5	36.49	37.51	0.50	1.01	0.55	1.00
-0.1	2.730	0.00	0.00	1.01	1.00	1.0	9.03	8.95	1.01	1.02	1.11	1.01
-0.1	2.730	0.00	0.00	1.01	1.00	2.0	3.25	3.17	1.99	1.03	2.19	1.02
-0.1	2.730	0.00	0.00	1.01	1.00	3.0	2.08	2.04	3.02	1.03	3.32	1.02
-0.1	2.730	0.00	0.00	1.01	1.00	4.0	1.59	1.54	4.01	1.03	4.41	1.02
-0.3	2.340	0.00	0.00	1.05	1.00	0.0	362.32	370.37	0.00	1.05	0.00	1.00
-0.3	2.340	0.00	0.00	1.05	1.00	0.5	26.26	26.28	0.50	1.05	0.65	1.01
-0.3	2.340	0.00	0.00	1.05	1.00	1.0	7.10	6.66	1.01	1.07	1.31	1.01
-0.3	2.340	0.00	0.00	1.05	1.00	2.0	2.85	2.61	2.00	1.09	2.60	1.02
-0.3	2.340	0.00	0.00	1.05	1.00	3.0	1.89	1.75	3.02	1.08	3.92	1.03
-0.3	2.340	0.00	0.00	1.05	1.00	4.0	1.46	1.24	4.01	1.07	5.21	1.01
-0.5	2.000	0.00	0.00	1.16	1.00	0.0	362.32	370.37	0.00	1.16	0.00	1.00
-0.5	2.000	0.00	0.00	1.16	1.00	0.5	22.39	19.45	0.50	1.16	0.75	1.01
-0.5	2.000	0.00	0.00	1.16	1.00	1.0	6.40	5.30	1.01	1.18	1.51	1.02
-0.5	2.000	0.00	0.00	1.16	1.00	2.0	2.66	2.22	2.00	1.21	3.01	1.02
-0.5	2.000	0.00	0.00	1.16	1.00	3.0	1.84	1.51	3.02	1.20	4.52	1.01
-0.5	2.000	0.00	0.00	1.16	1.00	4.0	1.44	1.08	4.01	1.19	5.99	1.02
-0.7	1.705	0.00	0.00	1.41	1.00	0.0	364.96	370.37	0.00	1.41	0.00	1.00
-0.7	1.705	0.00	0.00	1.41	1.00	0.5	24.07	14.52	0.50	1.41	0.85	1.00
-0.7	1.705	0.00	0.00	1.41	1.00	1.0	6.60	4.44	1.00	1.43	1.70	1.02
-0.7	1.705	0.00	0.00	1.41	1.00	2.0	2.75	1.98	2.00	1.47	3.42	1.02
-0.7	1.705	0.00	0.00	1.41	1.00	3.0	1.91	1.27	3.02	1.49	5.11	1.01
-0.7	1.705	0.00	0.00	1.41	1.00	4.0	1.55	1.02	4.01	1.46	6.78	1.02

3.6.1. MA(1) Model - Shift in Mean due to input :

The quality characteristic is modelled as

$$y(t) = z(t) - \theta * z(t-1)$$

where θ is a constant and $z(t) \sim NID(0,1)$.

If the shift in the mean of the process is due to the increase in the input mean of the process then

$$z'(t) = z(t) + \mu$$

and

$$z'(t) \sim NID(\mu, 1)$$

So

$$y'(t) = z'(t) - \theta * z'(t-1)$$

But we know that $z'(t) \sim NID(\mu, 1)$ and $\theta * z'(t-1) \sim NID(\mu\theta, 1)$. Since y (t) is correlated its distribution is determined by simulation . Upon simulation the steady state mean of the y(t) will be (μ (1-θ)) and will achieve this value on the second sample following the shift. Its standard deviation σ_y will remain unchanged in the steady state. For example for θ = 0.7 and a shift of μ=4.0 in the z's transforms the distribution of z's from NID(0,1) to NID(4,1) [See Figure-1 & Table-1]. This results in the transformation of the distribution of y(t) from (Mean = 0.0, S.D. = 1.22) before the shift to (1.2,1.22) after the shift [See Figure-2 & Table-1]. From this it can be seen that a 4σ shift in z(t) produces approximately a 1σ_y shift in y(t). However the control limits for the charts are not the same. The kσ limits used for plotting the y's and the z's are different. Whereas a value of k=3.0 is

Figure - 1 MA(1) Model

Shift=4.0 (Input) Lambda=0.25 THETA=0.7

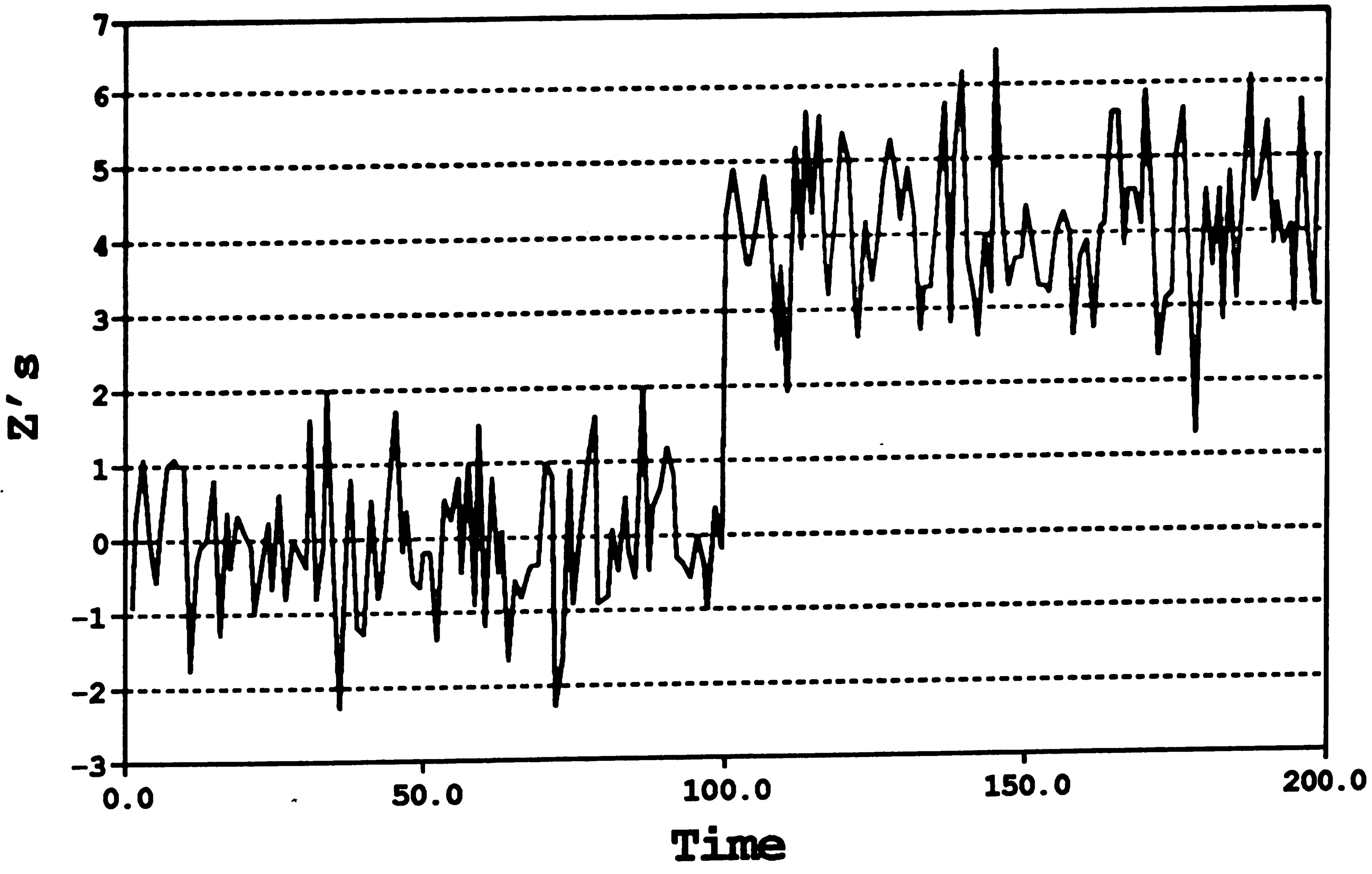
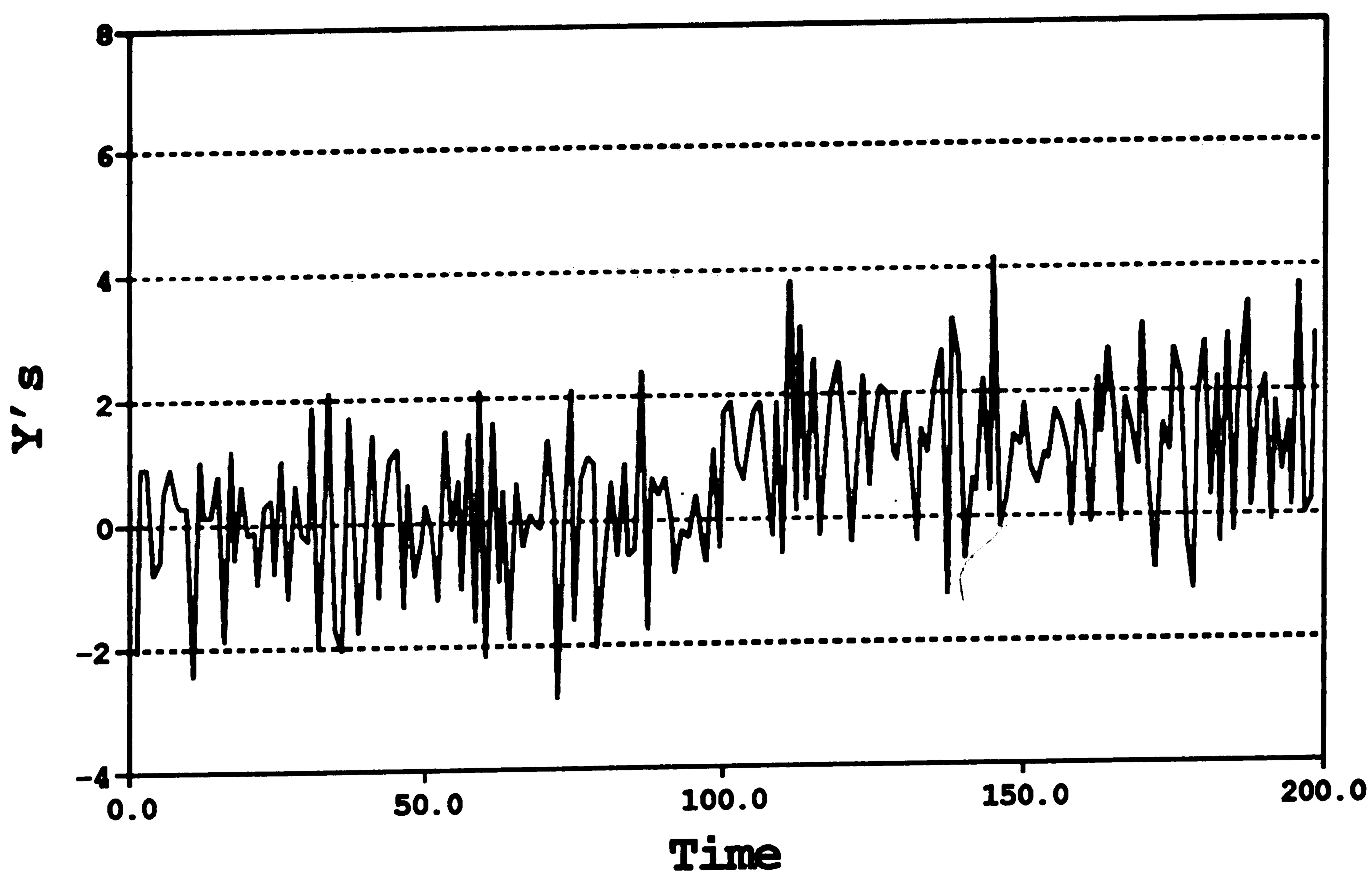


Figure - 2 MA(1) Model

Shift=4.0(Input) Lambda=0.25 THETA=0.7



used for the chart to plot the z's a value of k=1.68 is used when plotting the y's for this example . At these values of k the probability of generating a false alarm is the same for both the charts. The z chart have broader control limits than y chart and the ratio of the control limits vis-a-vis the ratio of the magnitude of the shifts in terms of standard deviation units is an important point to consider. For example

$$\text{Shift in z's / Shift in y's} = 4\sigma / 1\sigma = 4.0$$

$$\text{Control Limit factor ratio} = 2.97/1.68 = 1.75$$

Since 4.0 >> 1.75 it can be guessed that inspite of the broader control limits for z's they can detect the shifts faster because the ratio of the shifts is much higher than the ratio of the control limits. In other words it can be seen why the average run lengths are smaller for the z's than for the y's. This explains why plotting the residuals leads to a faster detection of the shift in the mean due to the input. Of course since the y's are correlated, this argument does not constitute a proof. However, the simulation results agree closely with this intuitive analysis. These results are further illustrated by the figures which plot the values of y and z before and after the shift in the mean of the process. The shift in the z's is more pronounced relative to the variability than the shift in the y's and so the probability of detecting a shift is higher . The variance of the z's is also lower than the y's. If however $\theta = -0.7$ then 4σ shift in z(t) [See Figure-1 & Table-1] produces a $6.8\sigma_y$ $[\mu\ (1-\theta)]$ shift in y(t) [See Figure-3& Table-1]. Here again, the control limits are different .

$$\text{Shift in y's / Shift in z's} = 5.7\sigma / 4\sigma = 1.4$$

$$\text{Control Limit factor ratio} = 2.97/1.68 = 1.75$$

Here the two ratios are much closer. Their average run lengths are also closer to each other.

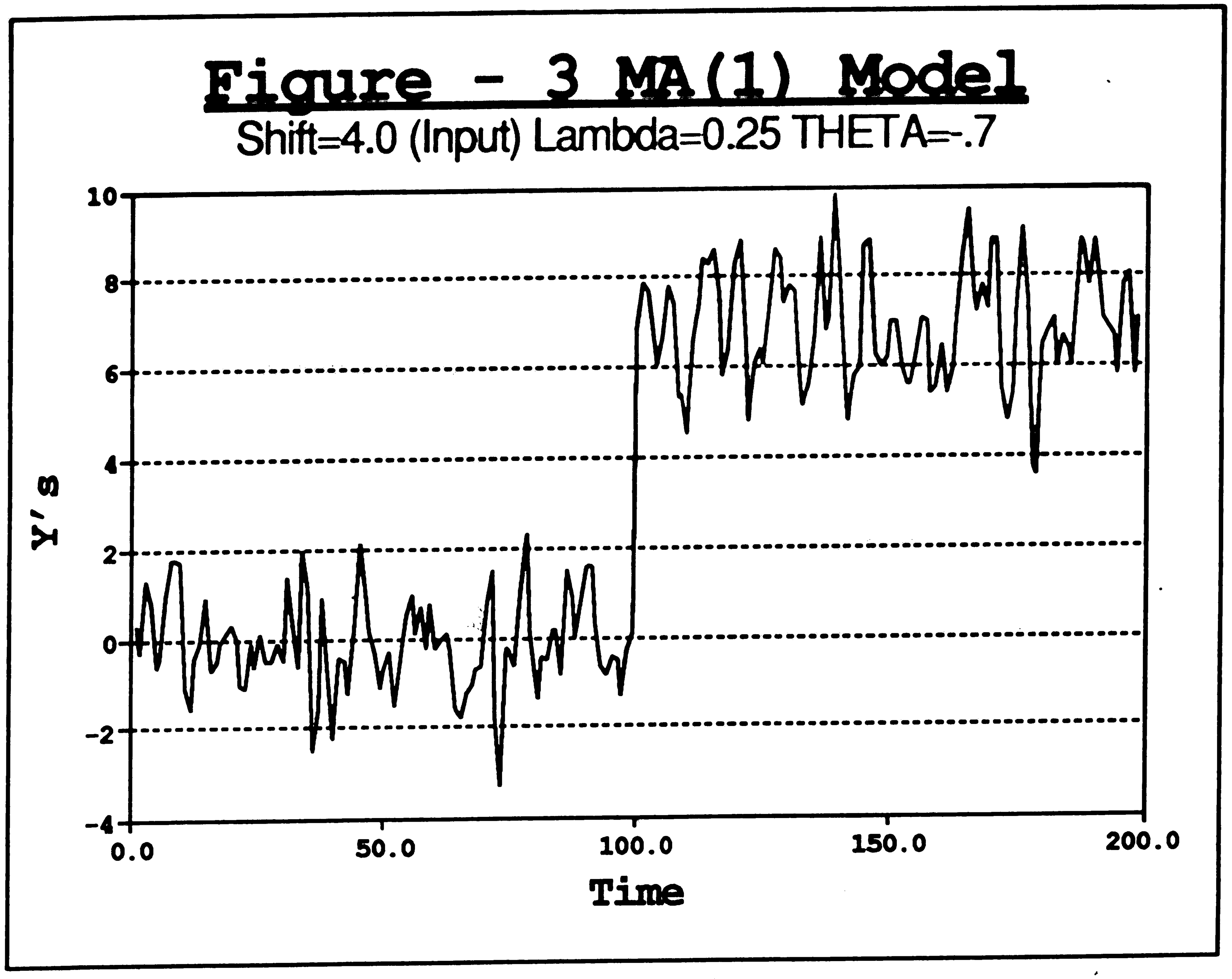


Figure - 3 MA(1) Model
Shift=4.0 (Input) Lambda=0.25 THETA=-.7
10
8
6
4
2
0
-2
-4
Y's
0.0
50.0
100.0
150.0
200.0
Time

3.6.2 MA(1) Model - Shift in Mean due to increase in process output:

The quality characteristic is modelled as

$$y(t) = z(t) - \theta * z(t-1)$$

where θ is a constant and $z(t) \sim NID\ (0,1)$ and the distribution of the y's upon simulation is a mean of 0 and standard deviation σ_y. This is before a shift.

Suppose that a shift in mean ocurs at the output. That is, for a shift of μ, the output can be expressed as

$$y(t) = z'(t) - \theta * z'(t-1) + \mu(t)$$

$$z(t) \sim NID\ (0,\sigma^2)$$

Let t_0 denote the time of the shift so that

$$\mu(t) = 0 \text{ when } t < t_0$$

$$\mu(t) = \mu \text{ when } t > t_0$$

The values of z to be plotted on the control chart are calculated from the filtered observations. Let z'(t) denote the filtered output. The effect of the shift on the mean of z'(t) can be calculated as follows :

$$E[z'(t_0 - 1)] = 0$$

$$E[z'(t_0)] = \theta * E[z'(t_0 - 1)] + E[y(t_0)] = \mu$$

$$E[z'(t_0 + 1)] = \theta * E[z'(t_0)] + \mu = (1+\theta)\ \mu$$

$$E[z'(t_0 + 2)] = \theta * E[z'(t_0 + 1)] + \mu = (1+\theta+\theta^2)\ \mu$$

$$E[z'(t_0 + k)] = \mu \sum_{i=0}^{k} \theta^k$$

In steady state as k goes to infinity, the mean of the z's approaches $\frac{\mu}{(1-\theta)}$.

For example, an MA(1) process with $\theta = 0.7$ and a 4.0 shift in mean at the output will produce a steady state shift of 13.32 in the z's. Upon simulation the distribution of the y's have transformed from (0,1.22) to a distribution for a shift in mean =4.0 and standard deviation = 1.22 [See Figure-4 & Table-2]. The z's upon introduction of the shift have been transformed from a NID(0,1) to a distribution whose mean = 13.32 and S.D. = 1.00 [See Figure-5 & Table-2] whenShift in z's / Shift in y's = 13.32σ /3.27 σ = 4.07

Control Limit factor ratio = 2.99/2.25 = 1.32

Since 4.07 > 1.32 the average run length for z's is shorter than the y's.

If θ = -0.7, then for a shift in mean = 4.0 of the y's [See Figure-6 & Table-2] the z's have been transformed from a NID(0,1) to a (1.24,1.03) [See Figure-7 & Table-2] distribution.

Shift in z's / Shift in y's =(4.0/2.3)σ /(1.24/1.03)σ = 1.41

Control Limit factor ratio = 3.78/2.97 = 1.30

Since 1.41 is almost equal to 1.30 their average run lengths are also nearly equal.

3.6.3. AR(1) Model - Shift in Mean at the input :

The quality characteristic or the output is modelled as

$$y(t) = \phi * y(t-1) + z(t)$$

where ϕ is a constant and $z(t) \sim NID(0,1)$

If the shift in the mean of the process is due to the increase at the process input then

$$z'(t) = z(t) + \mu$$

and

$$z'(t) \sim NID(\mu, 1)$$

Figure - 4 MA(1) Model

Shift=4.0(Output) Lambda=0.25 THETA=0.7

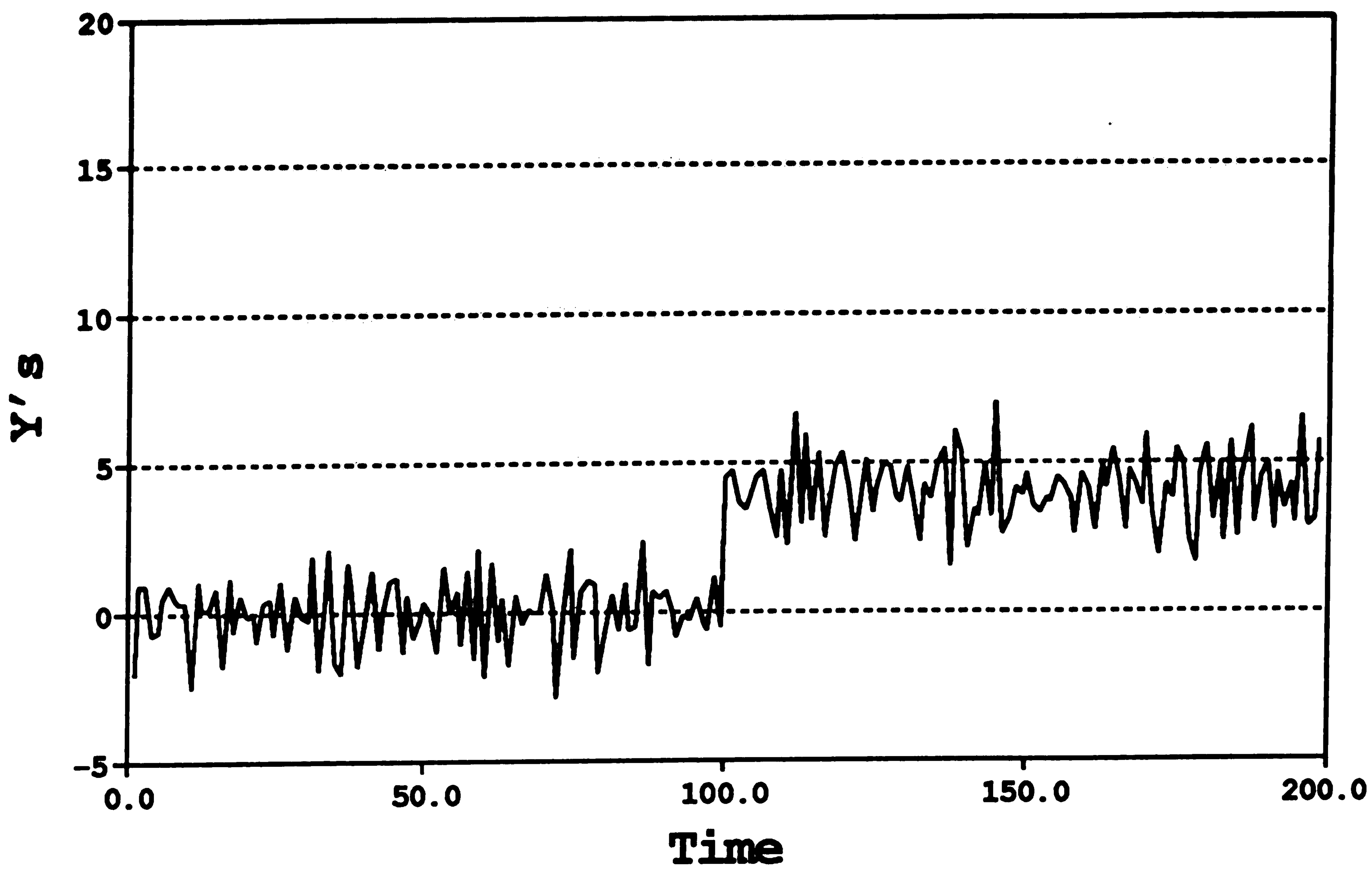


Figure - 5 MA(1) Model

Shift=4.0(Output) Lambda=0.25 THETA=0.7

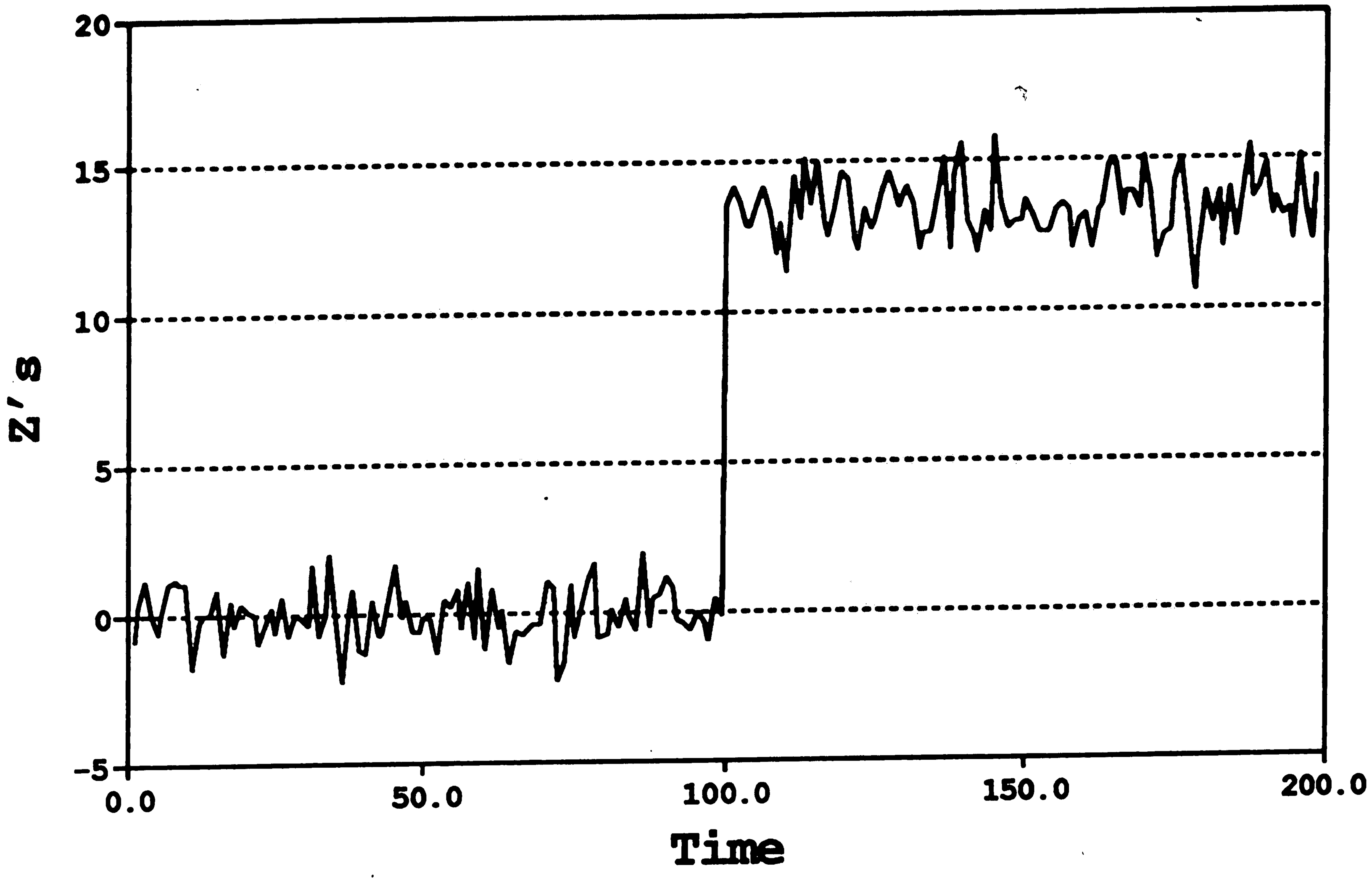


Figure - 6 MA(1) Model

Shift=4.0(Output) Lambda=0.25 THETA=-.7

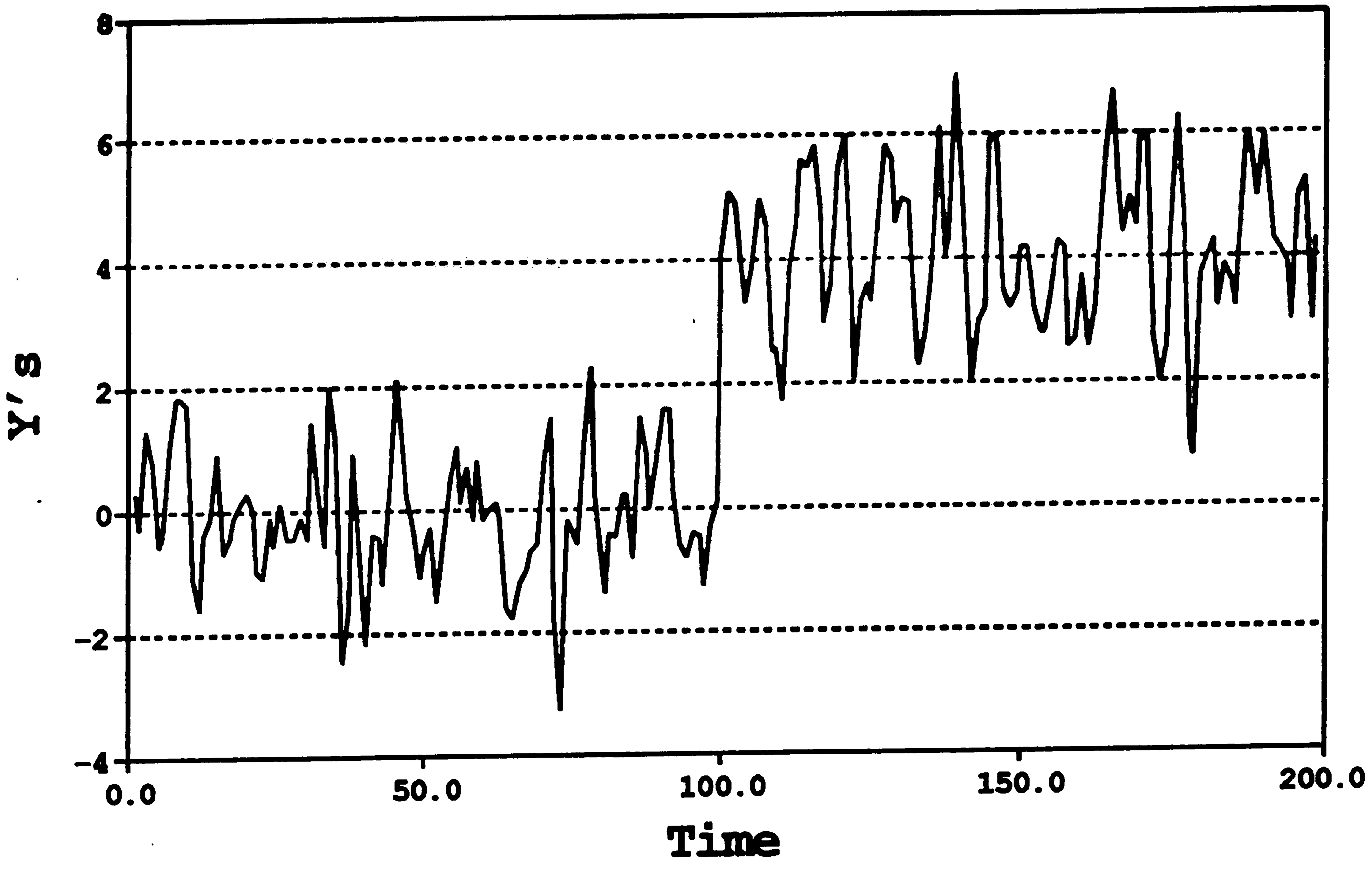
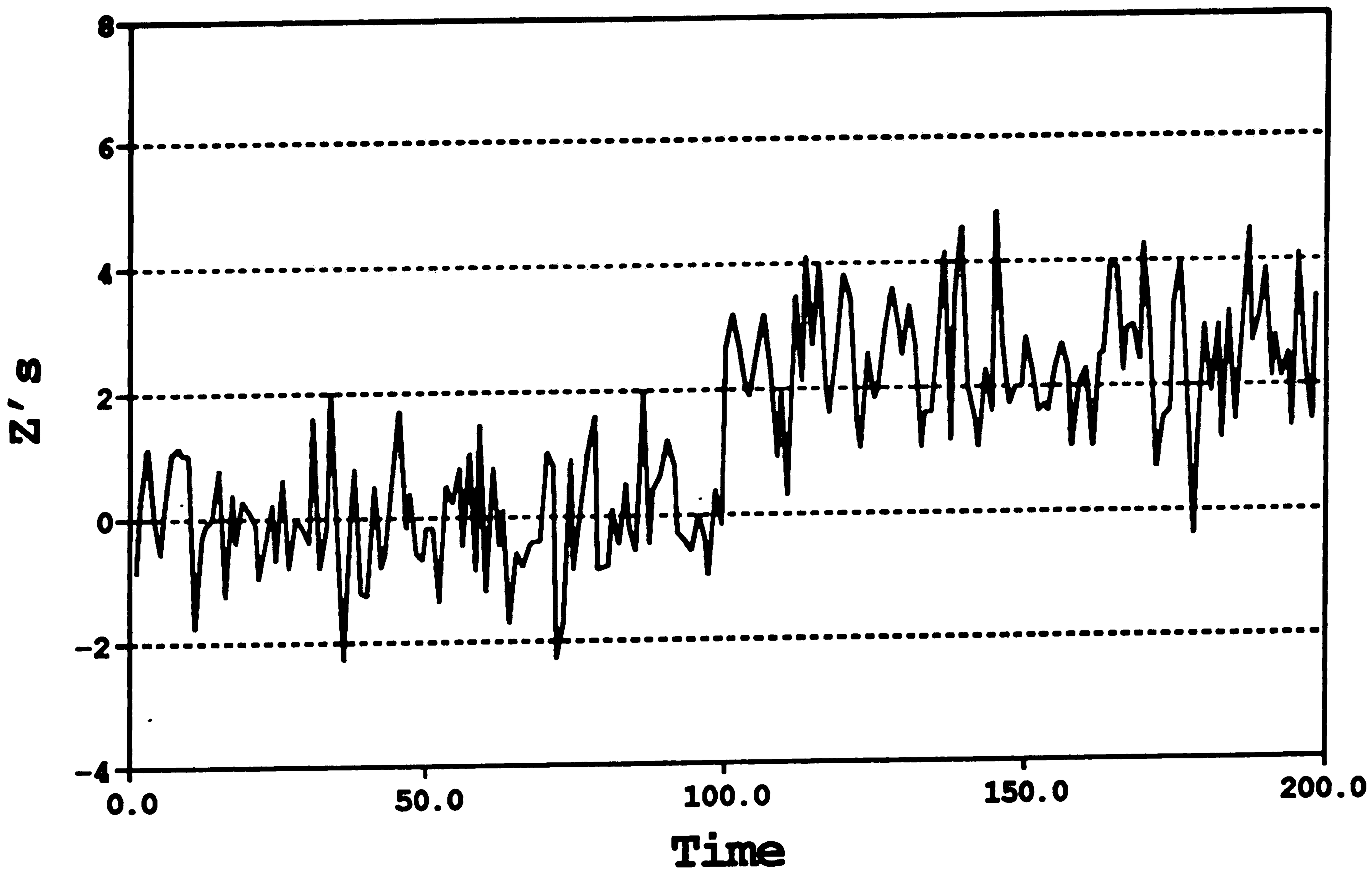


Figure - 7 MA(1) Model

Shift=4.0(Output) Lambda=0.25 THETA=-.7



So

$$y'(t) = \phi * y'(t-1) + z'(t)$$

But we know that $z'(t) \sim NID(\mu, 1)$ and $\theta * z'(t-1) \sim NID(\mu\theta, 1)$. Since y (t) is correlated its distribution is determined by simulation . The distribution of y (t) had to be determined using simulation because of the presence of serial correlation. For example for $\phi = 0.7$ and a shift of $\mu=4.0$ in the z's transforms the distribution of z's from NID(0,1) to NID(4,1) [See Figure-8 & Table-3]. This results in the transformation of the distribution of y(t) from (Mean = 0.0, S.D. = 1.41) before the shift to (13.4,1.41) [See Figure-9 & Table-3] after the shift. From this it can be seen that a 4σ shift in z(t) producess a $9.5\sigma_y$ shift in y(t). However the control limits for the charts are not the same. The y chart have broader control limits than z chart and the ratio of the control limits vis-a-vis the ratio of the magnitude of the shifts in terms of standard deviation units is important. For example

Shift in y's / Shift in z's = $9.5\sigma / 4.0\sigma = 2.35$

Control Limit factor ratio = 4.13/3.00 = 1.38

Since 2.35> 1.38 it would appear that inspite of the broader control limits for y's they can detect the shifts faster. In other words it can be seen as to why the average run lengths are smaller for the y's than for the z's. This explains why plotting the residuals leads to a faster detection of the shift in the mean at the input. These results are further corroborated by simulation and are illustrated by f igures which plot the values of y and z before and after the shift in the mean of the process. The shift in the y's is more pronounced than the shift in the z's and so the probability of detecting a shift is higher . If however $\theta = -0.7$ then 4σ shift in z(t) [See Figure-8 & Table-3] produces a $2.35/1.41=1.66\ \sigma_y$ shift in y(t) [See Figure-10& Table-3]. Here again, the control limits are different but the control limits for

Figure - 8 AR(1) Model

Shift=4.0(Input) Lambda=0.25 PHI=0.7

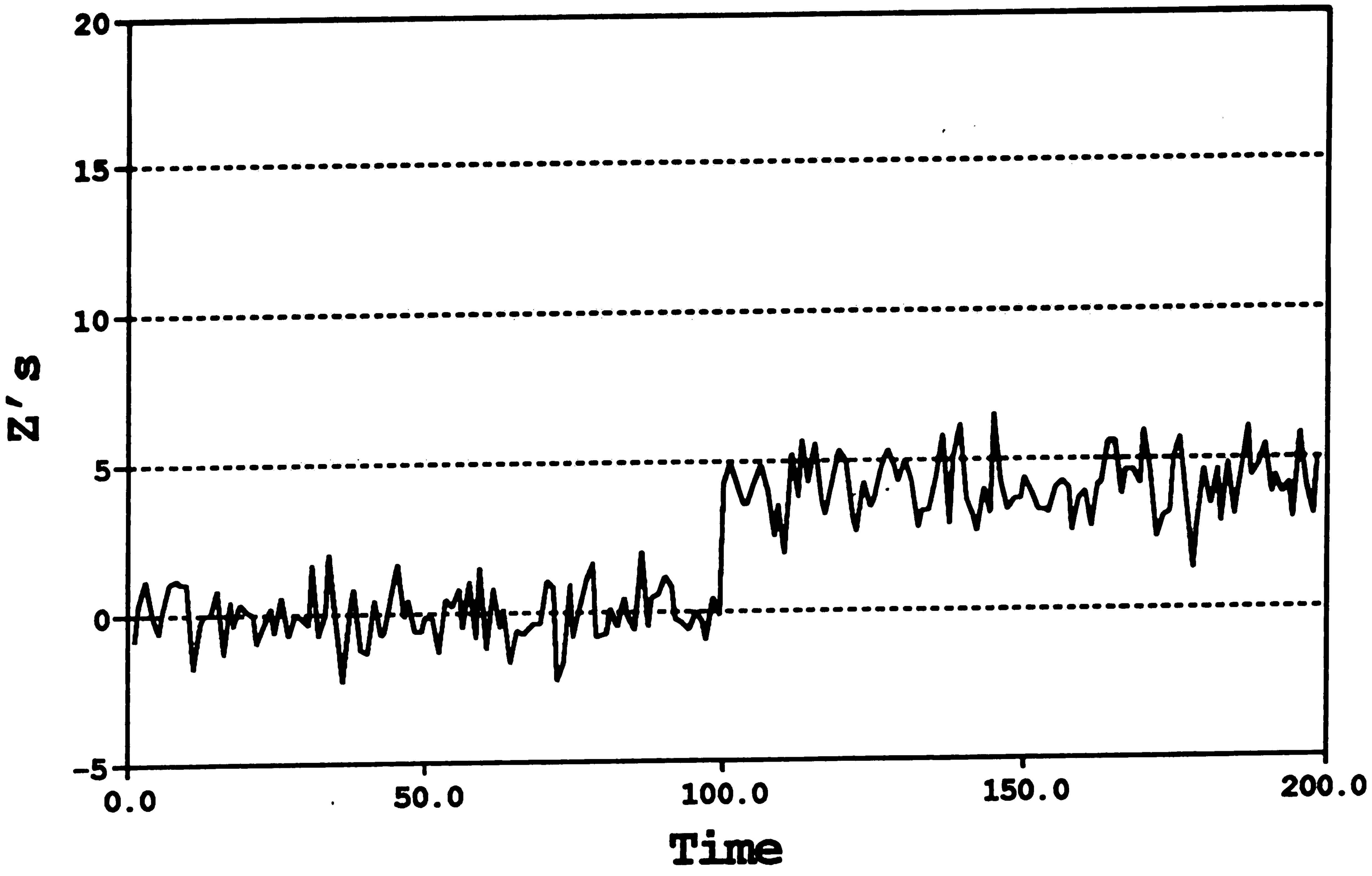


Figure - 9 AR(1) Model

Shift=4.0(Input) Lambda=0.25 PHI=0.7

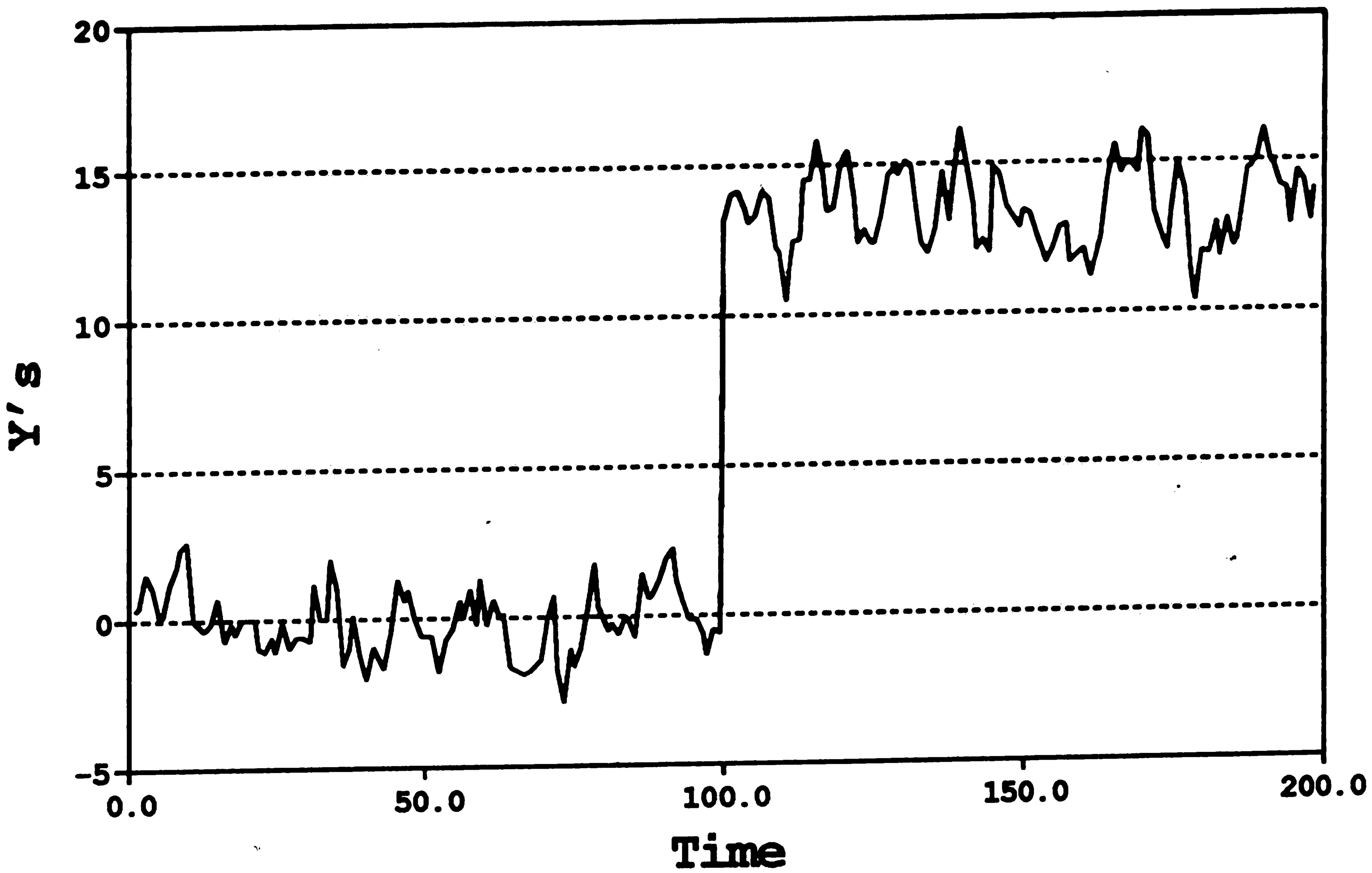
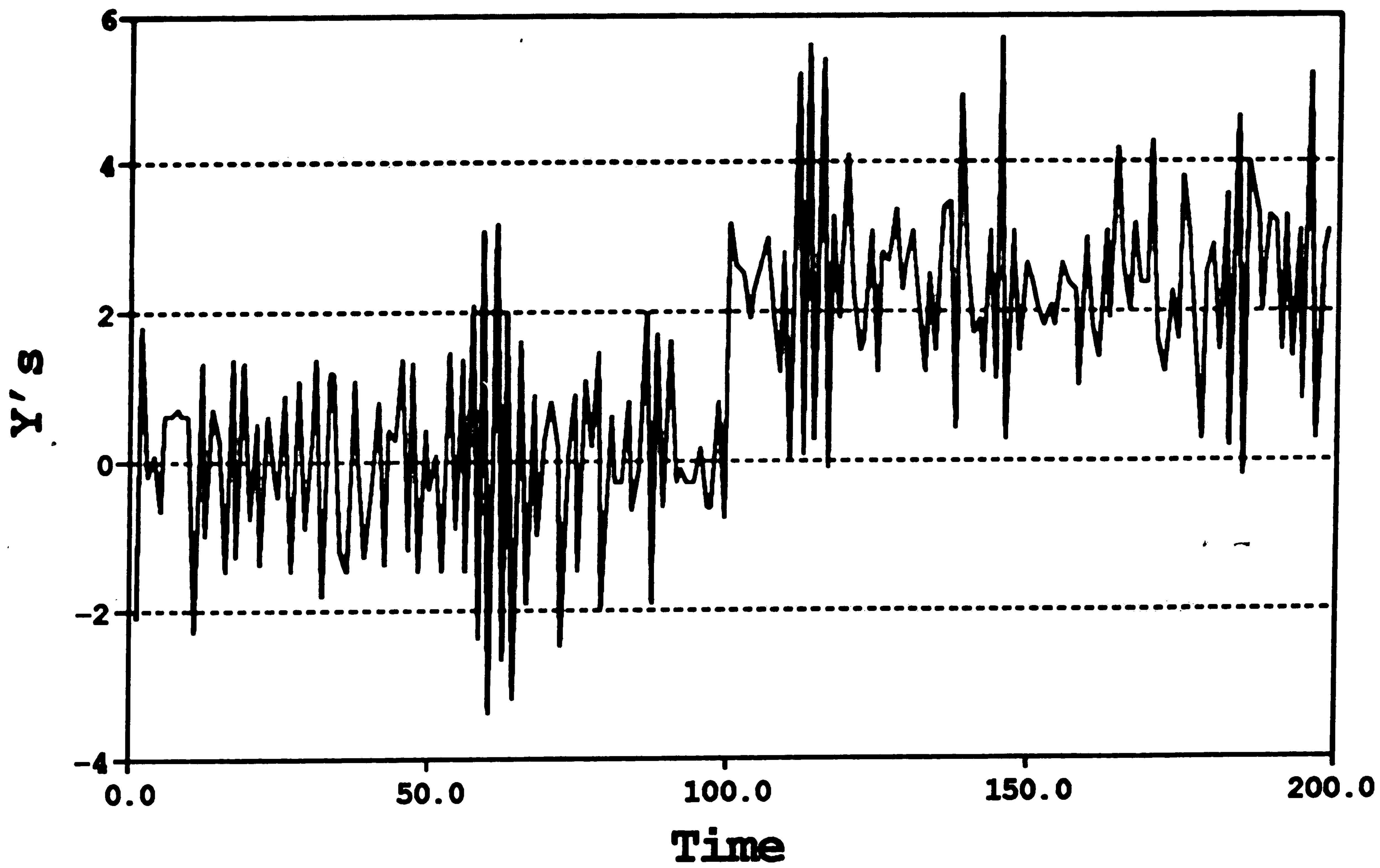


Figure - 10 AR(1) Model

Shift=4.0(Input) Lambda=0.25 PHI=-0.7



the y's are broader than that for the z's in addition to the magnitude of its shift being smaller than that of the z. So it is quite obvious in this case that that z's will detect a shift faster.

3.6.4. AR(1) Model - Shift in Mean due to increase in process output:

The quality characteristic or the observations is modelled as

$$y(t) = \phi * y(t-1) + z(t)$$

where ϕ is a constant and z(t)~NID(0,1). The distribution of the y (t) is determined by simulation and has a mean of 0.0 and standard deviation of 1.41 before the shift. Suppose that a shift in mean occurs at the output. That is, for a shift of μ, the output can be expressed as

$$y(t) = \phi * y(t-1) + \mu + z(t)$$

$$z(t) \sim NID\,(0, \sigma^2)$$

Let t_0 denote the time of the shift so that

$$\mu(t) = 0 \text{ when } t < t_0$$

$$\mu(t) = \mu \text{ when } t > t_0$$

The values of z to be plotted on the control chart are calculated from the filtered observations. Let z'(t) denote the filtered output. The effect of the shift on the mean of z'(t) can be calculated as follows :

$$E[z'(t_0 - 1)] = 0$$

$$E[z'(t_0)] = E[y(t_0)] - \phi * E[y(t_0 - 1)]$$

$$= E[y(t_0 + \mu] - E[\phi\,[y(t_0 - 1) + \mu\,]]$$

$$= E[z(t_0)] + \mu\,(1-\phi)$$

$$E[z'(t_0 + 1)] = E[z(t_0 + 1)] + \mu\,(1-\phi)$$

$$E[z'(t_0 + k)] = E[z(t_0+k)] + \mu\,(1-\phi)$$

The steady state is reached after the first observation after the shift and it takes a constant form thereafter.For example, an MA(1) process with $\theta = 0.7$ and a 4.0 shift in mean at the output will produce a steady state shift of 1.20 in the z's. For a shift of $\mu=4.0$ the distribution of the y's have transformed from (0,1.41) to a distribution for a shift in mean =4.0 and standard deviation = 1.41[See Figure-11& Table-4] . The z's upon introduction of the shift have been transformed from a NID(0,1) to a distribution whose mean = 1.20 and S.D. = 1.00[See Figure-12& Table-4]

Shift in y's / Shift in z's = $(4.0/1.41)\sigma$ /$(1.2/1.0)\ \sigma$ =2.35

Control Limit factor ratio = 4.88/3.00 = 1.63

Since 2.35 > 1.63 the average run length for y's is shorter than the z's.

If on the other hand $\phi = -0.7$ then y's transform from (0,1.41) to a (4.0,1.41) distribution [See Figure-13 & Table-4] whereas the z's transform from a NID(0,1) to (6.78,1)[See Figure-14 & Table-4] but the control limits for the z's are wider.

Shift in z's / Shift in y's = $(6.78/1.0)\ \sigma\ (4.0/1.41)\sigma$ =2.38

Control Limit factor ratio = 4.88/3.00 = 1.63

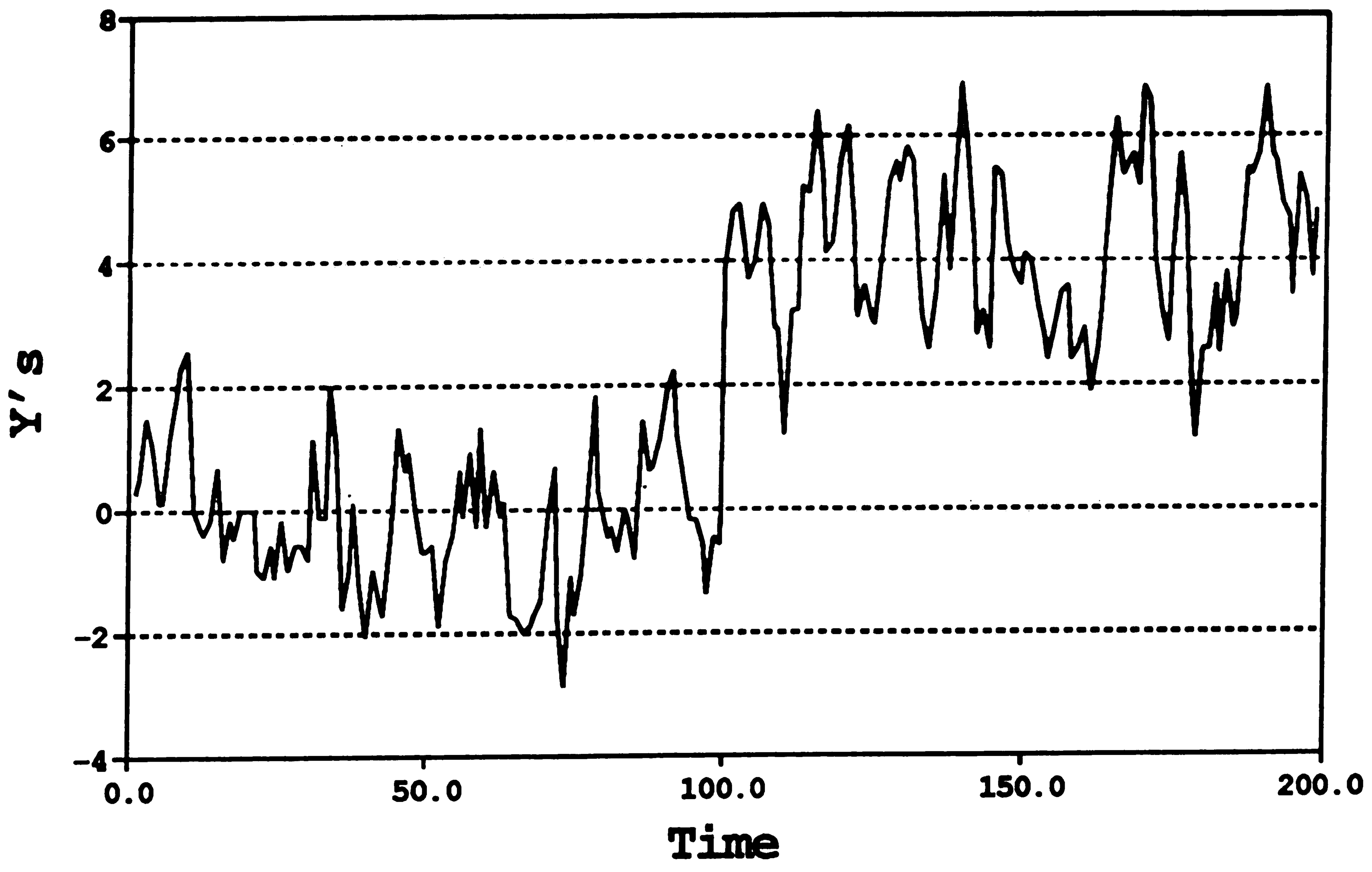
Since 2.38 > 1.63 the average run length for z's is shorter than the y's.

3.7.0 Motivation for an Alternate Approach:

Upon observing the results of the first approach certain interesting observations were

Figure - 11 AR(1) Model

Shift=4.0(Output) Lambda=0.25 Phi=0.7



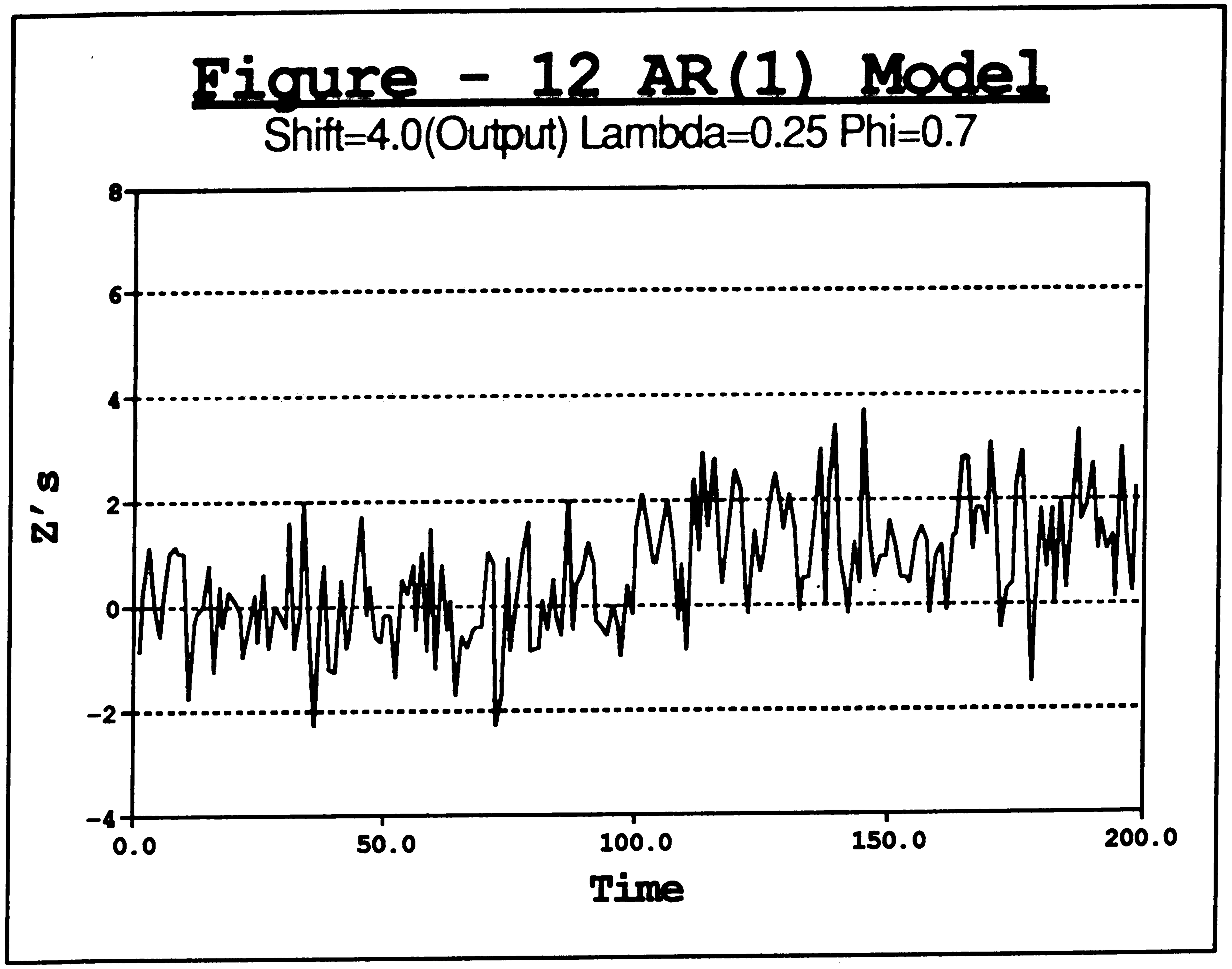


Figure - 12 AR(1) Model
Shift=4.0(Output) Lambda=0.25 Phi=0.7
8
6
4
2
0
-2
-4
Z's
0.0
50.0
100.0
150.0
200.0
Time

Figure - 13 AR(1) Model

Shift=4.0(Output) Lambda=0.25 PHI=-0.7

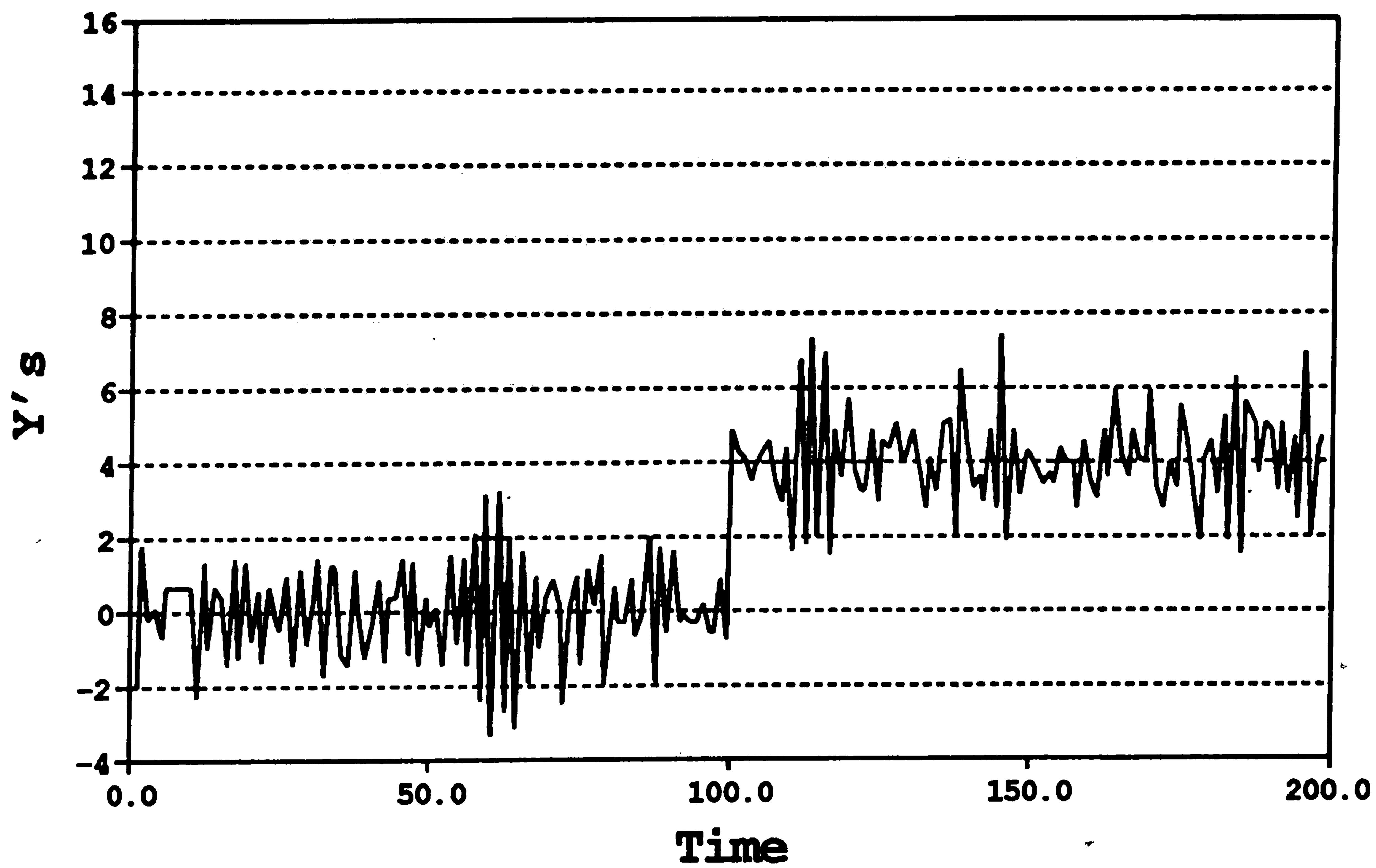
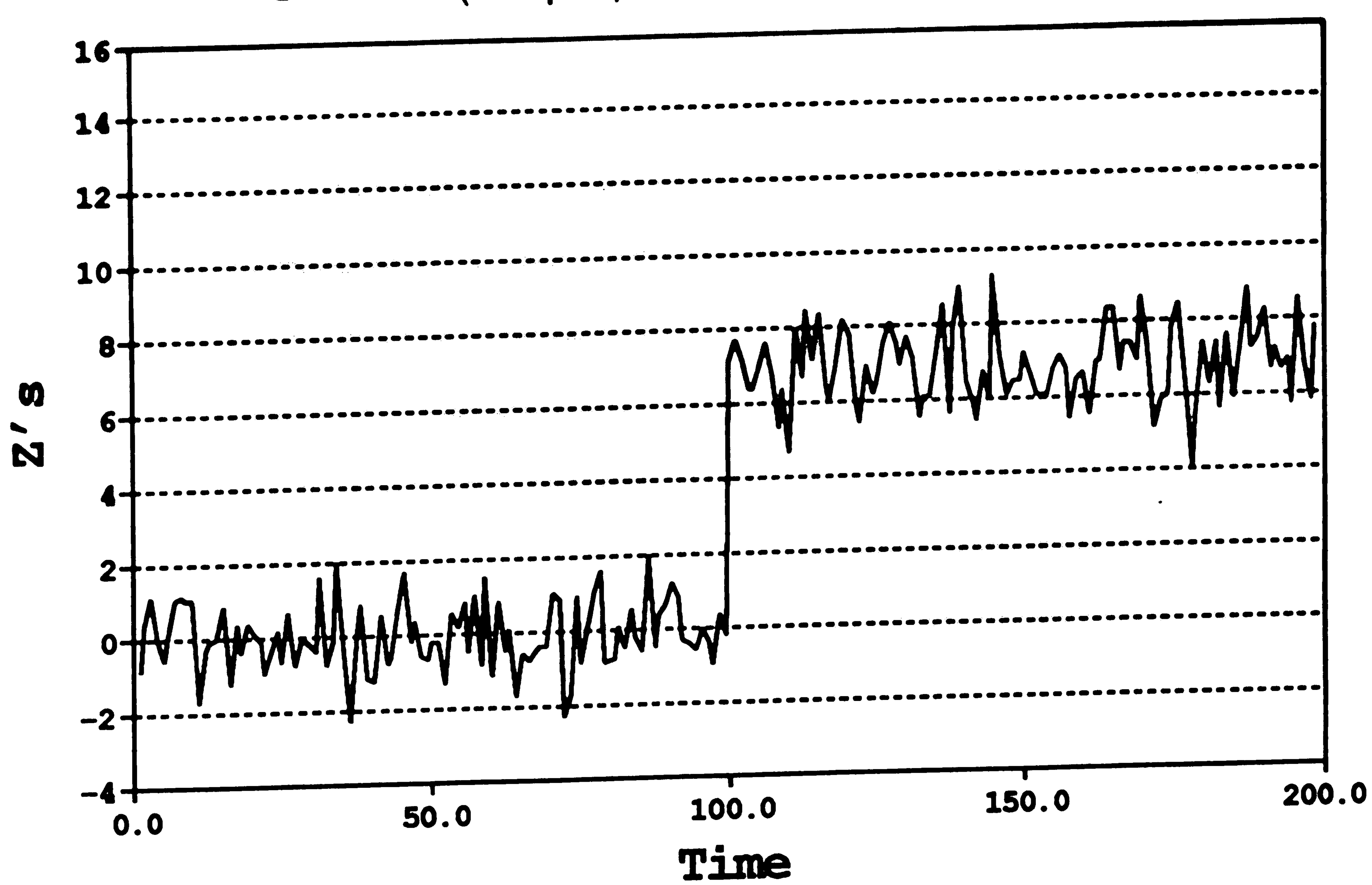


Figure - 14 AR(1) Model

Shift=4.0(Output) Lambda=0.25 PHI=-0.7



made. Our objective of proposing the first approach was to develop a statistical procedure to detect shifts at both the input and output stages of a correlated process. The proposed approach was to model the correlative structure as a time series model, use that model to remove the autocorrelation and apply control charts to the residuals . For comparison purposes we also investigated the usual approach of plotting a control chart with the process output y(t).

Figure - 15 shows the comparison of the Average Run Lengths (ARL) between a MA(1) and an AR(1) model for different values of the ARMA paramaters θ or ϕ. The average run lengths of a GMA chart on the z(t) are also plotted in this figure. The assignable cause responsible for the shift was due to a step change in the input or z(t). For example if the process output could be modelled as an AR(1) process, then the quality characteristic could be represented as

$$y(t) = \phi * y(t-1) + z(t)$$

$$z(t) = y(t) - \phi\, y(t-1)$$

$$z(t) \sim NID\,(0, \sigma^2)$$

before the shift and

$$y'(t) = \phi\, y'(t-1) + \mu + z(t)$$

$$z'(t) = y'(t) - \phi\, y'(t-1)$$

after the shift where

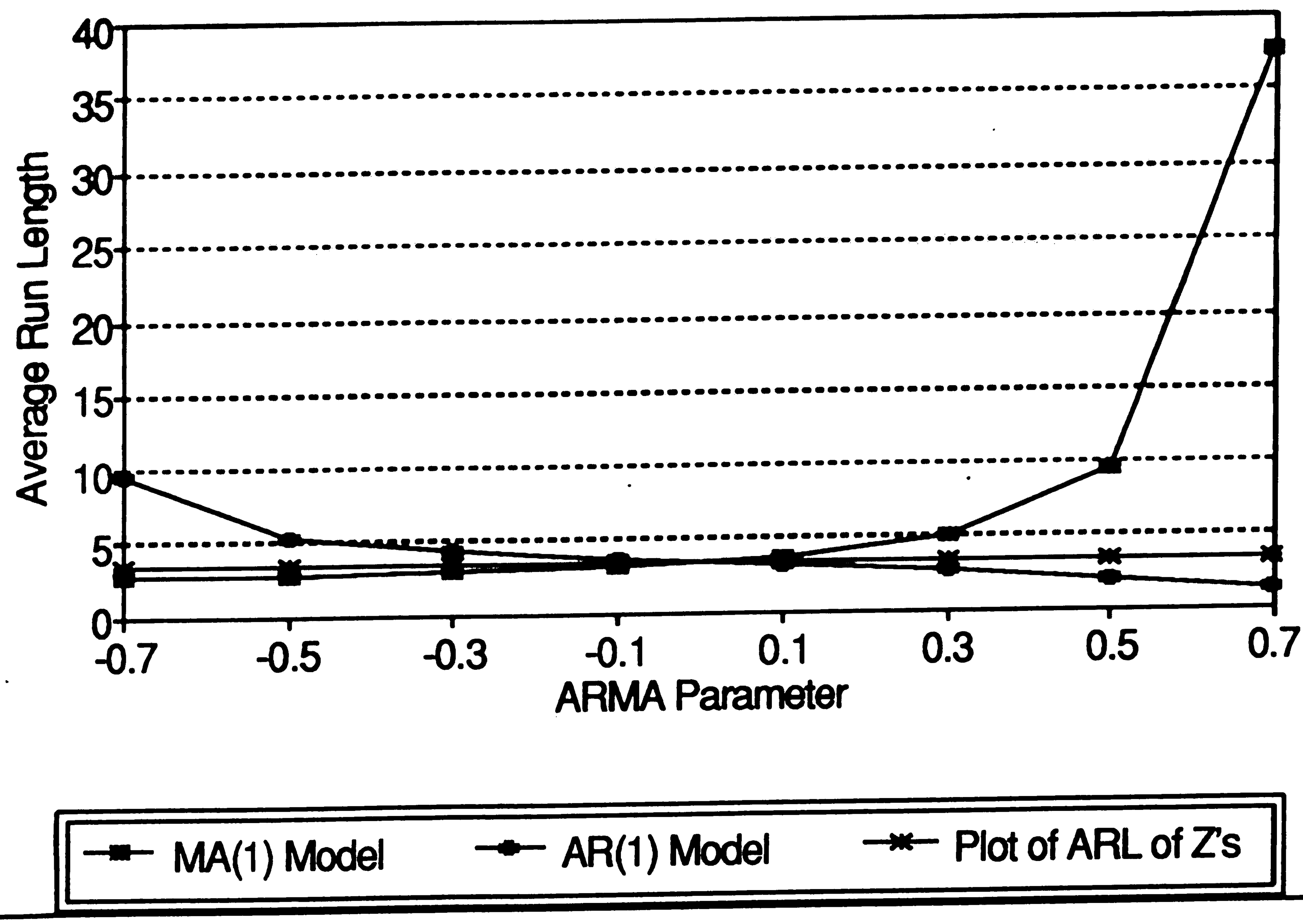
$$z'(t) = z(t) + \mu$$

$$z'(t) \sim NID\,(\mu, \sigma^2))$$

Likewise for an MA(1) process the quality characteristic or the process output can be

Figure-15 MA(1) Model Vs AR(1) Model

Shift = 2.0 (Input) Lambda = 0.50



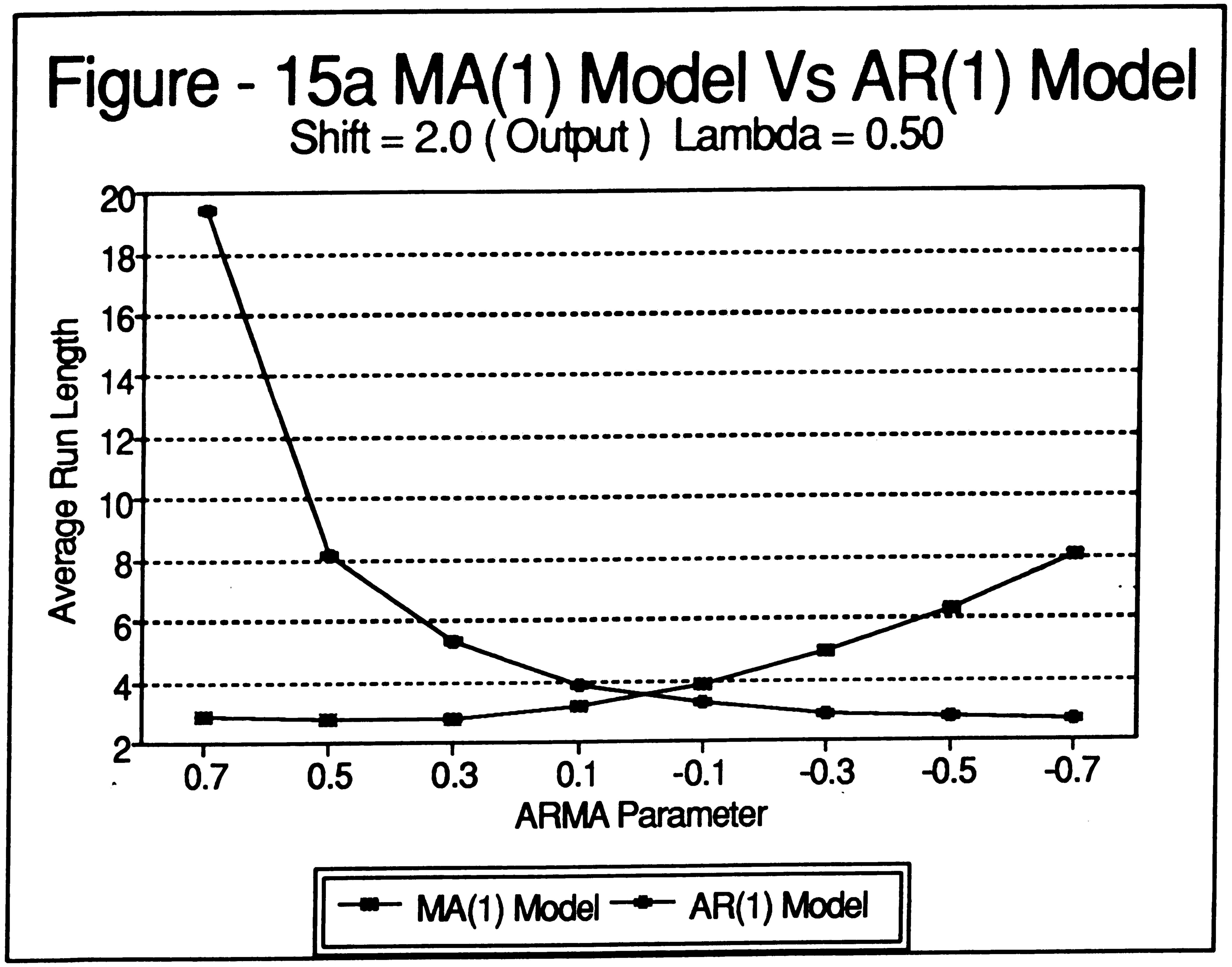


Figure - 15a MA(1) Model Vs AR(1) Model
Shift = 2.0 (Output) Lambda = 0.50
Average Run Length
20
18
16
14
12
10
8
6
4
2
0.7
0.5
0.3
0.1
-0.1
-0.3
-0.5
-0.7
ARMA Parameter
MA(1) Model
AR(1) Model

modelled as

$$y(t) = z(t) - \theta * z(t-1)$$

$$z(t) = y(t) + \theta\, z(t-1)$$

$$z(t) \sim NID\,(0,\sigma^2)$$

before the shift and

$$y'(t) = z(t) + \mu - \theta *[\, z(t-1) + \mu]$$

$$z'(t) = y'(t) + \theta * z'(t-1)$$

after the shift where

$$z'(t) = z(t) + \mu$$

The y's and the z's are plotted on a Geometric Moving Average Chart to determine the ARLs . Figure - 15 plots the values of the ARL obtained by simulation for AR(1) and MA(1) process for a specified shift in the input and GMA parameter ($\lambda = 0.5$). The Figure also plots the ARLs of the uncorrelated z's . It can be seen that the ARL of z's is a constant for all values of the ARMA parameter. This is due to the fact that the z's are uncorrelated and the ARMA parameter has no effect whatsoever on its ARL.

On the other hand the ARMA parameter has a strong influence on the ARL of y's . An ideal situation would be for the ARL of z's to be smaller than the ARL of y's whenever a shift occured and under all environments. It is however not always the case. It can be seen from the Figure that for an MA(1) model the ARL of z's are smaller than the ARL of y's for $\theta > 0.0$ and vice versa for $\theta < 0$. The opposite is true for an AR(1) process. So if the model under consideration was an MA(1) process whose ARMA parameter was less than 0.0 and a shift in the mean of the process was due to the shift in the input [z(t)], then plotting the z(t) would detect shifts slower as opposed to the y's. However if the model under consideration was an AR(1) process whose parameter was greater than 0.0 and a shift in the mean of the process was due to the shift in the input [z(t)],then plotting the $\tilde{z}(t)$ would detect shifts faster as opposed to the y's. It is clear that the results from an MA(1) and an AR(1) processes behave like mirror images of each other when the shift occurs at the input.

Figure - 16 shows the comparison of the ARL between an AR(1) and an MA(1) process for different values of the ARMA parameter when the shift in the mean of the process is due to a step change in the process output.

Figures 16 and 17shows the comparison of the ARL's of the y's and z's for AR(1) and MA(1) models respectively.

It is apparent that the "best procedure" depends on the particular ARMA process and on where the shift occurs. The observance of this behaviour is what led us to the second approach where it is proposed to take advantage the mirroring effect of the two time series models.

Figure-16 ARL(Y) Vs ARL(Z)-MA(1) Model

Shift = 2.0 (Output) Lambda = 0.50

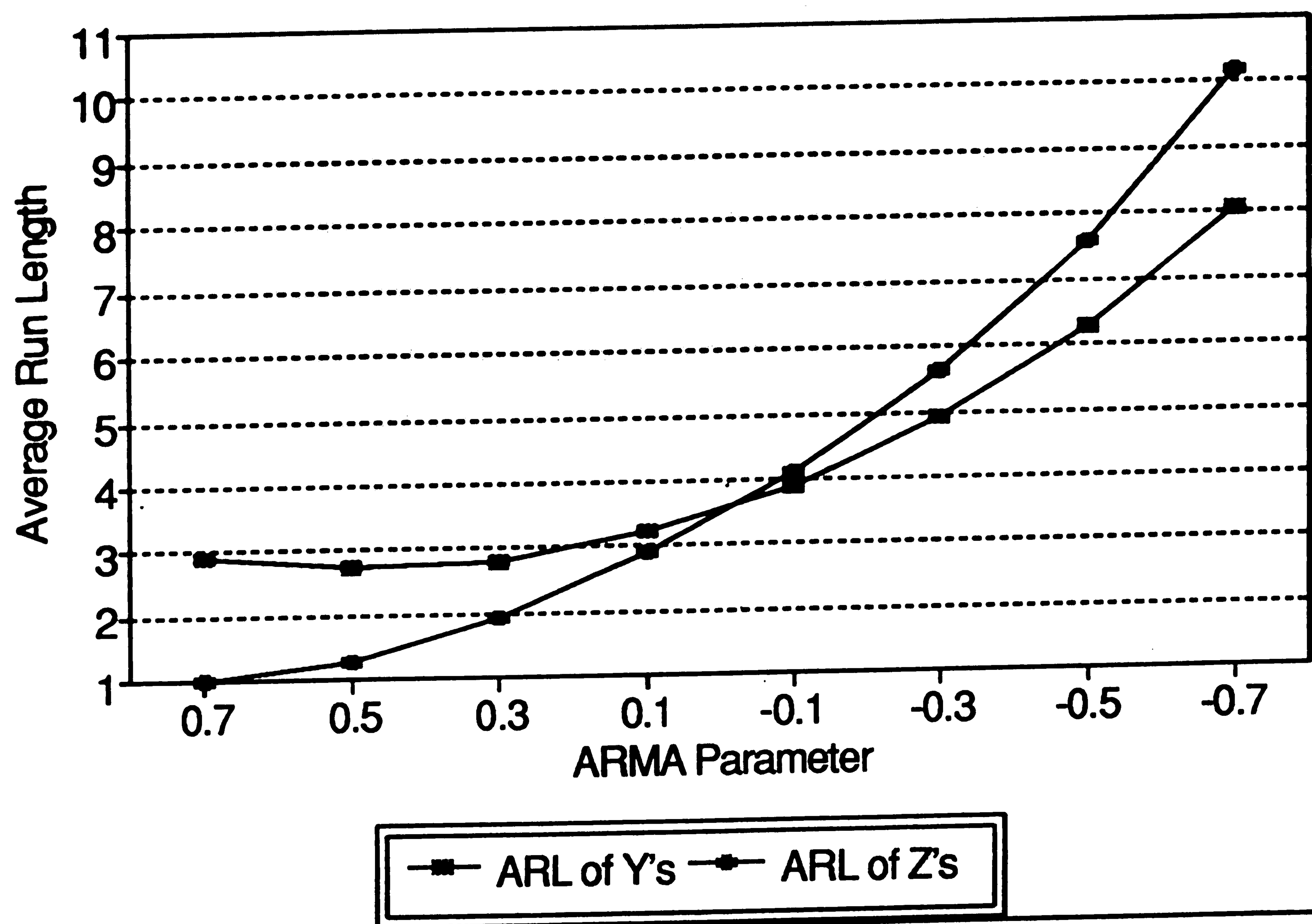
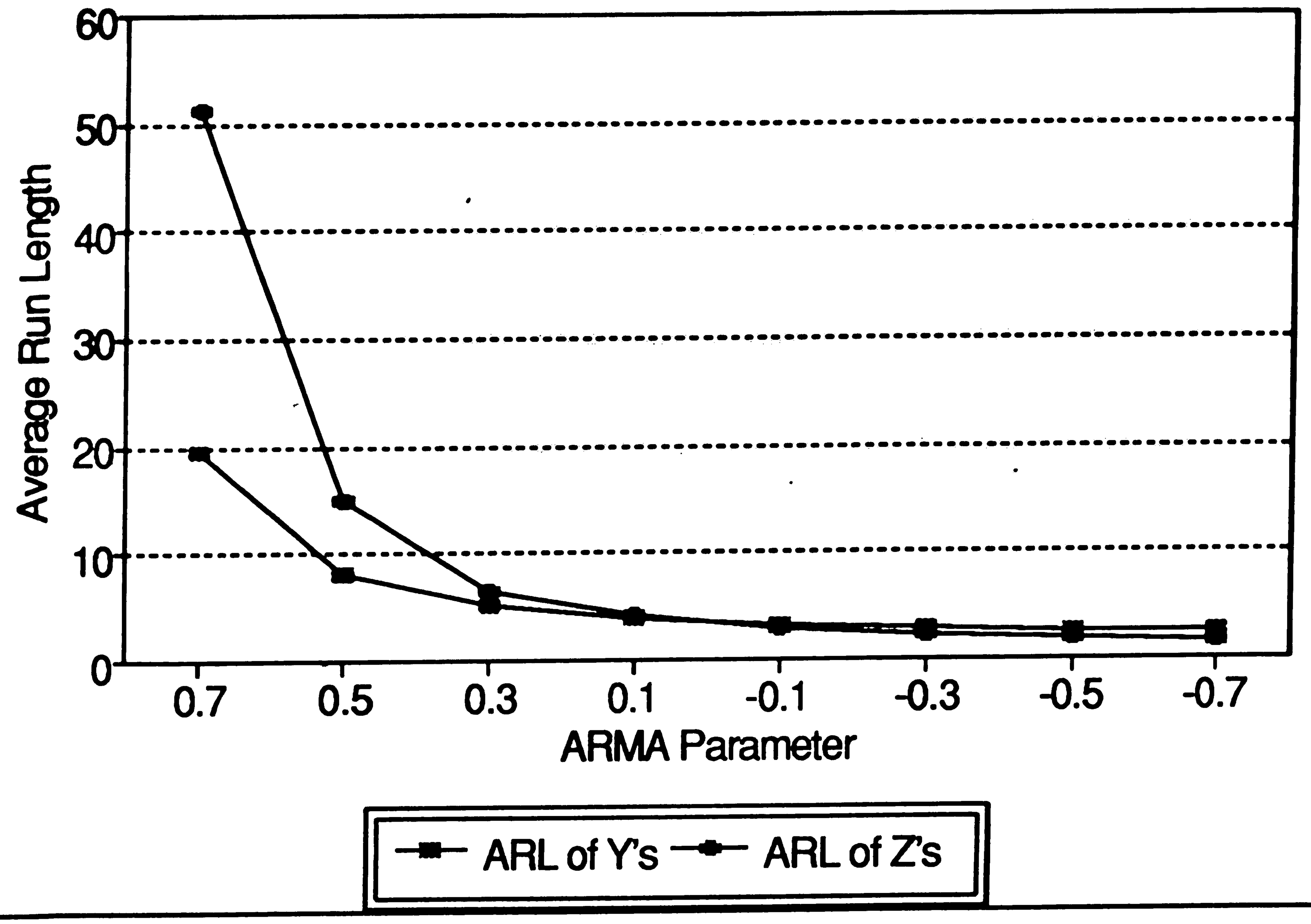


Figure-17 ARL(Y) Vs ARL(Z)- AR(1) Model

Shift = 2.0 (Output) Lambda = 0.50



4.0 Alternate Approach:

The earlier approach was to model the correlative structure of a process as a time series model , use that model to remove the autocorrelation from the data and apply control charts to the residuals. The alternate or the second approach goes a step further. Due to the results of the first experiment it is proposed to transform the uncorrelated residuals or random noise to an alternate time series model and apply control charts on the transformed data . For example if the manufacturing process under consideration was an MA(1) model then the process output or the quality characteristic can be represented as

$$y(t) = z(t) - \theta * z(t-1)$$

$$z(t) = y(t) + \theta\, y(t - 1)$$

$$z(t) \sim NID\,(0, \sigma^2)$$

before the shift and

$$y'(t) = z(t) + \mu - \theta *[\, z\,(t-1) + \mu\,]$$

$$z'(t) = y'(t) + \theta * z\,'(t-1$$

after the shift where

$$z'(t) = z(t\,) + \mu$$

$$z(t) \sim NID\,(\mu, \sigma^2)\,)$$

If the ARMA parameter $\theta > 0.0$ we know from Figure -1 that ARL of y's for an AR(1) process is better than the ARL of y's from a MA(1) process and the ARL of the z's . So we transform the residuals [z(t)] from an MA(1) model to a variable say x(t) such that x(t) represents an AR(1) process . In other words,

$$x(t) = \varphi * x(t-1) + z'(t)$$

where $z(t)$ are the back filtered residuals

and $\phi = \theta$ for a MA(1) process

This is representative of a first order autoregressive model . The time series x(t) does not actually represent the process output but is a manipulated variable to take advantage of the superiority of the AR(1) model in detecting shifts faster . The values of x(t) are then plotted on a control chart and their average run lengths determined by simulation. If on the other hand the ARMA parameter for the MA(1) model $\theta < 0.0$ then such a transformation is not required because the ARLof y(t) obtained from the MA(1) model is the fastest in detecting the shift.

Likewise if the process could be modelled in such a way that its process output or quality characteristic can be represented as an AR(1) model, then

$$y(t) = \phi * y(t-1) + z(t)$$

$$z(t) = y(t) - \phi\, y(t-1)$$

$$z(t) \sim NID\ (0,\sigma^2)$$

before the shift and

$$y'(t) = \phi\, y'(t-1) + \mu + z(t)$$

$$z'(t) = y'(t) - \phi\, y'(t-1)$$

after the shift in z.

If the ARMA parameter $\phi < 0.0$ we know from Figure -1 that ARL of y's for a MA(1) process is better than the ARL of y's from an AR(1) process and the ARL of the z's. So we transform the z(t) the residuals from an AR(1) model to a variable say x(t) such that x(t) represents an MA(1) process . In other words,

$$x(t) = z'(t) - \theta * z'(t-1)$$

where $z'(t)$ are the back filtered residuals

and $\theta = \phi$ for an AR(1) process

This is representative of a first order moving average model. The time series x(t) does not actually represent the process output but is a manipulated variable used to take advantage of the superiority of the MA(1) model in detecting shifts faster under the circumstances of the ARMA parameter being less than zero. The values of x(t) are then plotted on a control chart and their average run lengths determined.

If on the other hand the ARMA parameter for the AR(1) model $\theta > 0.0$ then such a transformation is not required because the ARL of y(t) obtained from the AR(1) model is the fastest in detecting the shift in the mean of the process.

The alternate approach just described considered shifts in the mean of the process due to a step change in the input of the process. The same approach can be used for detecting shifts in the mean of the process due to process outputs also.

If the manufacturing process could be modelled in such a way that its process output or quality characteristic can be represented as an MA(1) model and a shift in the mean of the process is due to a step change in the proecess output, then

$$y(t) = z(t) - \theta * z(t-1)$$

$$z(t) = y(t) + \theta * z(t-1)$$

$$z(t) \sim NID\ (0,\sigma^2))$$

before the shift and

$$y'(t) = y(t) + \mu$$

$$z'(t) = \theta * z'(t-1) + y'(t)$$

after the shift.

Since the process is MA(1), a transformation of z(t) if required would be to an AR(1) process such as

$$x(t) = \phi * x(t-1) + z'(t)$$

where $\phi = \theta$ for a MA(1) process

Likewise if the process could be modelled in such a way that its process output or quality characteristic can be represented as an AR(1) model and a shift in the mean of the process is due to a step change in the proecess output, then

$$y(t) = \phi * y(t-1) + z(t)$$

$$z(t) = y(t) - \phi * y(t-1)$$

$$z(t) \sim NID\ (0, \sigma^2))$$

before the shift and

$$y'(t) = \phi\ y'(t-1) + z'(t) = \phi\ y(t-1) + \mu + z'(t)$$

$$z'(t) = y'(t) - \phi\ y'(t-1)$$

after the shift.

Since the process is AR(1), a transformation of z(t) if required would be to a MA(1) process such as

$$x(t) = z'(t) - \theta * z'(t-1)$$

The approach just described is based on observation of the first experiment. It ignores the possibility that undesirable transient effects may be induced in the mean of x(t). However, the first experiment revealed that these transients are short lived, and had little effect on the ARL.

4.1.0 Results:

A simulation model was necessary to test the second approach since it was not possible to determine the Average Run Lengths analytically. The simulation model built for the first approach was extended to include the second approach. The factors used for experimentation of this approach were exactly the same for the first approach as were the levels and environments in which they were simulated.

The results are shown in Tables 5 - 8. These tables show the control limit factors used for the x's, model parameters, the mean and standard deviation of the transfomed data [x(t)] both before and after the shift , the in control alpha , the average run length and the magnitude of the shift in the process. The results are discussed in the following sections.

4.1.1.MA(1) Model - Shift in mean due to input:

When the process model is MA(1) and the shift occurs at the input,the ARL's of x(t) [See Table-5] behaves the same as like the ARL's of y(t) for an AR(1) process[See Figure-9 & Table-3] and the arguments pertaining to the AR(1) model whose shift in mean was at the input apply here as well . In other words,

$$x(t) = \varphi * x(t-1) + z'(t) = y'(t) \text{ for an AR(1) process}$$

4.1.2 MA(1) Model - Shift in mean due to step change in process output:

Here the process model is MA(1) and the shift occurs at the output. The distributions of

TABLE 5

Residuals of MA(1) Model Transformed to an AR(1)Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.97**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAX=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE SHIFT***	Sd.X ***BEFORE SHIFT***	SHIFT	ARLX	Av.X ****AFTER SHIFT****	Sd.X ****AFTER SHIFT****
0.9	5.720	-0.01	2.33	0.0000	362.319	-0.0132	2.3319
0.9	5.720	-0.01	2.33	0.5000	10.659	5.0137	2.3671
0.9	5.720	-0.01	2.33	1.0000	3.018	9.8891	2.2686
0.9	5.720	-0.01	2.33	2.0000	1.407	20.0650	2.1866
0.9	5.720	-0.01	2.33	3.0000	1.005	29.7565	2.1192
0.9	5.720	-0.01	2.33	4.0000	1.004	39.7698	2.1195
0.7	4.880	0.00	1.41	0.0000	367.647	-0.0043	1.4137
0.7	4.880	0.00	1.41	0.5000	26.922	1.6571	1.4323
0.7	4.880	0.00	1.41	1.0000	6.014	3.3520	1.4209
0.7	4.880	0.00	1.41	2.0000	2.258	6.6858	1.3866
0.7	4.880	0.00	1.41	3.0000	1.550	10.0429	1.3857
0.7	4.880	0.00	1.41	4.0000	1.012	13.2469	1.3621
0.5	4.180	0.00	1.16	0.0000	367.647	-0.0026	1.1608
0.5	4.180	0.00	1.16	0.5000	35.279	0.9959	1.1662
0.5	4.180	0.00	1.16	1.0000	7.873	2.0254	1.1805
0.5	4.180	0.00	1.16	2.0000	2.719	4.0002	1.1810
0.5	4.180	0.00	1.16	3.0000	1.858	6.0524	1.1657
0.5	4.180	0.00	1.16	4.0000	1.239	8.0103	1.1599
0.3	3.630	0.00	1.05	0.0000	373.134	-0.0018	1.0510
0.3	3.630	0.00	1.05	0.5000	40.372	0.7099	1.0543
0.3	3.630	0.00	1.05	1.0000	9.147	1.4379	1.0629
0.3	3.630	0.00	1.05	2.0000	3.110	2.8382	1.0695
0.3	3.630	0.00	1.05	3.0000	2.013	4.3119	1.0593
0.3	3.630	0.00	1.05	4.0000	1.480	5.7293	1.0499
0.1	3.170	0.00	1.01	0.0000	370.370	-0.0014	1.0062
0.1	3.170	0.00	1.01	0.5000	43.698	0.5540	1.0080
0.1	3.170	0.00	1.01	1.0000	10.196	1.1149	1.0139
0.1	3.170	0.00	1.01	2.0000	3.395	2.2106	1.0252
0.1	3.170	0.00	1.01	3.0000	2.144	3.3487	1.0255
0.1	3.170	0.00	1.01	4.0000	1.655	4.4622	1.0332

TABLE 5

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	2.730	0.00	1.01	0.0000	362.319	-0.0012	1.0059
-0.1	2.730	0.00	1.01	0.5000	44.782	0.4533	1.0075
-0.1	2.730	0.00	1.01	1.0000	10.727	0.9095	1.0121
-0.1	2.730	0.00	1.01	2.0000	3.662	1.8194	1.0281
-0.1	2.730	0.00	1.01	3.0000	2.287	2.7319	1.0314
-0.1	2.730	0.00	1.01	4.0000	1.745	3.6587	1.0398
-0.3	2.340	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.340	0.00	1.05	0.5000	45.789	0.3830	1.0522
-0.3	2.340	0.00	1.05	1.0000	11.403	0.7711	1.0546
-0.3	2.340	0.00	1.05	2.0000	3.938	1.5375	1.0764
-0.3	2.340	0.00	1.05	3.0000	2.455	2.3109	1.0866
-0.3	2.340	0.00	1.05	4.0000	1.850	3.0971	1.0820
-0.5	2.000	0.00	1.16	0.0000	362.319	-0.0009	1.1597
-0.5	2.000	0.00	1.16	0.5000	49.934	0.3326	1.1598
-0.5	2.000	0.00	1.16	1.0000	12.819	0.6667	1.1609
-0.5	2.000	0.00	1.16	2.0000	4.308	1.3334	1.1865
-0.5	2.000	0.00	1.16	3.0000	2.662	2.0017	1.2057
-0.5	2.000	0.00	1.16	4.0000	1.991	2.6824	1.2086
-0.7	1.705	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	1.705	0.00	1.41	0.5000	66.845	0.2933	1.4115
-0.7	1.705	0.00	1.41	1.0000	16.682	0.5876	1.4068
-0.7	1.705	0.00	1.41	2.0000	5.223	1.1799	1.4402
-0.7	1.705	0.00	1.41	3.0000	3.211	1.7595	1.4526
-0.7	1.705	0.00	1.41	4.0000	2.309	2.3558	1.4716

TABLE 5

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	2.730	0.00	1.01	0.0000	362.319	-0.0012	1.0059
-0.1	2.730	0.00	1.01	0.5000	44.782	0.4533	1.0075
-0.1	2.730	0.00	1.01	1.0000	10.727	0.9095	1.0121
-0.1	2.730	0.00	1.01	2.0000	3.662	1.8194	1.0281
-0.1	2.730	0.00	1.01	3.0000	2.287	2.7319	1.0314
-0.1	2.730	0.00	1.01	4.0000	1.745	3.6587	1.0398
-0.3	2.340	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.340	0.00	1.05	0.5000	45.789	0.3830	1.0522
-0.3	2.340	0.00	1.05	1.0000	11.403	0.7711	1.0546
-0.3	2.340	0.00	1.05	2.0000	3.938	1.5375	1.0764
-0.3	2.340	0.00	1.05	3.0000	2.455	2.3109	1.0866
-0.3	2.340	0.00	1.05	4.0000	1.850	3.0971	1.0820
-0.5	2.000	0.00	1.16	0.0000	362.319	-0.0009	1.1597
-0.5	[illegible].000	0.00	1.16	0.5000	49.934	0.3326	1.1598
-0.5	[illegible].000	0.00	1.16	1.0000	12.819	0.6667	1.1609
-0.5	2.000	0.00	1.16	2.0000	4.308	1.3334	1.1865
-0.5	2.000	0.00	1.16	3.0000	2.662	2.0017	1.2057
-0.5	2.000	0.00	1.16	4.0000	1.991	2.6824	1.2086
-0.7	1.705	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	1.705	0.00	1.41	0.5000	66.845	0.2933	1.4115
-0.7	1.705	0.00	1.41	1.0000	16.682	0.5876	1.4068
-0.7	1.705	0.00	1.41	2.0000	5.223	1.1799	1.4402
-0.7	1.705	0.00	1.41	3.0000	3.211	1.7595	1.4526
-0.7	1.705	0.00	1.41	4.0000	2.309	2.3558	1.4716

TABLE 6

Residuals of MA(1) Model Transformed to an AR(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.97**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAX=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	5.720	-0.01	2.33	0.0000	362.319	-0.0132	2.3319
0.9	5.720	-0.01	2.33	0.5000	1.009	49.7668	2.1219
0.9	5.720	-0.01	2.33	1.0000	1.006	99.8305	2.1223
0.9	5.720	-0.01	2.33	2.0000	1.004	199.9459	2.1303
0.9	5.720	-0.01	2.33	3.0000	1.003	300.0572	2.1457
0.9	5.720	-0.01	2.33	4.0000	1.003	400.1674	2.1655
0.7	4.880	0.00	1.41	0.0000	367.647	-0.0043	1.4137
0.7	4.880	0.00	1.41	0.5000	2.785	5.5353	1.4311
0.7	4.880	0.00	1.41	1.0000	1.255	11.1116	1.4004
0.7	4.880	0.00	1.41	2.0000	1.003	22.1457	1.3658
0.7	4.880	0.00	1.41	3.0000	1.002	33.2688	1.3668
0.7	4.880	0.00	1.41	4.0000	1.002	44.3923	1.3672
0.5	4.180	0.00	1.16	0.0000	367.647	-0.0026	1.1608
0.5	4.180	0.00	1.16	0.5000	7.873	2.0254	1.1805
0.5	4.180	0.00	1.16	1.0000	2.723	3.9992	1.1809
0.5	4.180	0.00	1.16	2.0000	1.241	8.0104	1.1592
0.5	4.180	0.00	1.16	3.0000	1.002	11.9514	1.1426
0.5	4.180	0.00	1.16	4.0000	1.002	15.9558	1.1426
0.3	3.630	0.00	1.05	0.0000	373.134	-0.0018	1.0510
0.3	3.630	0.00	1.05	0.5000	18.659	1.0132	1.0594
0.3	3.630	0.00	1.05	1.0000	4.953	2.0474	1.0624
0.3	3.630	0.00	1.05	2.0000	2.095	4.0976	1.0588
0.3	3.630	0.00	1.05	3.0000	1.339	6.1345	1.0573
0.3	3.630	0.00	1.05	4.0000	1.013	8.1294	1.0477
0.1	3.170	0.00	1.01	0.0000	370.370	-0.0014	1.0062
0.1	3.170	0.00	1.01	0.5000	34.712	0.6151	1.0115
0.1	3.170	0.00	1.01	1.0000	8.345	1.2436	1.0210
0.1	3.170	0.00	1.01	2.0000	2.988	2.4570	1.0296
0.1	3.170	0.00	1.01	3.0000	1.967	3.7307	1.0244
0.1	3.170	0.00	1.01	4.0000	1.452	4.9453	1.0090

TABLE 6

THET	Contrl Limit	Av.X	Sd.X	SHIFT	ARLX	Av.X	Sd.X
		***BEFORE	SHIFT***			****AFTER	SHIFT****
-0.1	2.730	0.00	1.01	0.0000	362.319	-0.0012	1.0059
-0.1	2.730	0.00	1.01	0.5000	53.533	0.4121	1.0059
-0.1	2.730	0.00	1.01	1.0000	12.806	0.8262	1.0095
-0.1	2.730	0.00	1.01	2.0000	4.194	1.6478	1.0253
-0.1	2.730	0.00	1.01	3.0000	2.544	2.4795	1.0358
-0.1	2.730	0.00	1.01	4.0000	1.891	3.3340	1.0327
-0.3	2.340	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.340	0.00	1.05	0.5000	73.099	0.2949	1.0504
-0.3	2.340	0.00	1.05	1.0000	18.990	0.5885	1.0544
-0.3	2.340	0.00	1.05	2.0000	5.521	1.1890	1.0730
-0.3	2.340	0.00	1.05	3.0000	3.289	1.7662	1.0765
-0.3	2.340	0.00	1.05	4.0000	2.387	2.3757	1.0864
-0.5	2.000	0.00	1.16	0.0000	362.319	-0.0009	1.1597
-0.5	2.000	0.00	1.16	0.5000	97.656	0.2213	1.1597
-0.5	2.000	0.00	1.16	1.0000	28.345	0.4440	1.1647
-0.5	2.000	0.00	1.16	2.0000	7.585	0.8973	1.1833
-0.5	2.000	0.00	1.16	3.0000	4.312	1.3336	1.1859
-0.5	2.000	0.00	1.16	4.0000	3.088	1.7685	1.1932
-0.7	1.705	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	1.705	0.00	1.41	0.5000	151.057	0.1722	1.4115
-0.7	1.705	0.00	1.41	1.0000	50.607	0.3452	1.4115
-0.7	1.705	0.00	1.41	2.0000	12.258	0.6911	1.4133
-0.7	1.705	0.00	1.41	3.0000	6.214	1.0430	1.4355
-0.7	1.705	0.00	1.41	4.0000	4.269	1.3832	1.4468

TABLE 7

Residuals of AR(1) Model Transformed to an MA(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.97**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAX=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE SHIFT***	Sd.X ***BEFORE SHIFT***	SHIFT	ARLX	Av.X ****AFTER SHIFT****	Sd.X ****AFTER SHIFT****
0.9	1.585	0.00	1.35	0.0000	370.370	-0.0002	1.3457
0.9	1.585	0.00	1.35	0.5000	301.205	0.0498	1.3457
0.9	1.585	0.00	1.35	1.0000	233.645	0.0998	1.3457
0.9	1.585	0.00	1.35	2.0000	90.909	0.1998	1.3457
0.9	1.585	0.00	1.35	3.0000	42.509	0.2998	1.3501
0.9	1.585	0.00	1.35	4.0000	23.126	0.3993	1.3473
0.7	1.680	0.00	1.22	0.0000	370.370	-0.0004	1.2210
0.7	1.680	0.00	1.22	0.5000	140.449	0.1496	1.2210
0.7	1.680	0.00	1.22	1.0000	41.115	0.2993	1.2247
0.7	1.680	0.00	1.22	2.0000	10.802	0.6005	1.2248
0.7	1.680	0.00	1.22	3.0000	5.963	0.9017	1.2429
0.7	1.680	0.00	1.22	4.0000	4.204	1.1992	1.2513
0.5	1.910	0.00	1.12	0.0000	364.964	-0.0007	1.1185
0.5	1.910	0.00	1.12	0.5000	68.306	0.2493	1.1185
0.5	1.910	0.00	1.12	1.0000	17.491	0.4985	1.1206
0.5	1.910	0.00	1.12	2.0000	5.658	1.0029	1.1407
0.5	1.910	0.00	1.12	3.0000	3.416	1.4959	1.1499
0.5	1.910	0.00	1.12	4.0000	2.491	2.0011	1.1614
0.3	2.290	0.00	1.04	0.0000	373.134	-0.0009	1.0446
0.3	2.290	0.00	1.04	0.5000	51.230	0.3491	1.0446
0.3	2.290	0.00	1.04	1.0000	12.739	0.6999	1.0450
0.3	2.290	0.00	1.04	2.0000	4.296	1.3998	1.0674
0.3	2.290	0.00	1.04	3.0000	2.616	2.1046	1.0776
0.3	2.290	0.00	1.04	4.0000	1.961	2.8167	1.0782
0.1	2.725	0.00	1.01	0.0000	367.647	-0.0012	1.0057
0.1	2.725	0.00	1.01	0.5000	45.981	0.4479	1.0073
0.1	2.725	0.00	1.01	1.0000	10.934	0.9005	1.0118
0.1	2.725	0.00	1.01	2.0000	3.709	1.7996	1.0291
0.1	2.725	0.00	1.01	3.0000	2.305	2.7037	1.0298
0.1	2.725	0.00	1.01	4.0000	1.756	3.6229	1.0379

TABLE 7

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	3.160	0.00	1.01	0.0000	373.134	-0.0014	1.0060
-0.1	3.160	0.00	1.01	0.5000	44.090	0.5490	1.0078
-0.1	3.160	0.00	1.01	1.0000	10.299	1.1028	1.0137
-0.1	3.160	0.00	1.01	2.0000	3.416	2.1905	1.0255
-0.1	3.160	0.00	1.01	3.0000	2.159	3.3136	1.0252
-0.1	3.160	0.00	1.01	4.0000	1.662	4.4180	1.0342
-0.3	3.460	0.00	1.05	0.0000	367.647	-0.0017	1.0453
-0.3	3.460	0.00	1.05	0.5000	42.198	0.6467	1.0479
-0.3	3.460	0.00	1.05	1.0000	9.782	1.3056	1.0549
-0.3	3.460	0.00	1.05	2.0000	3.303	2.5841	1.0627
-0.3	3.460	0.00	1.05	3.0000	2.076	3.9215	1.0547
-0.3	3.460	0.00	1.05	4.0000	1.566	5.2185	1.0617
-0.5	3.650	0.00	1.12	0.0000	370.370	-0.0019	1.1195
-0.5	3.650	0.00	1.12	0.5000	40.865	0.7457	1.1225
-0.5	3.650	0.00	1.12	1.0000	9.501	1.5066	1.1318
-0.5	3.650	0.00	1.12	2.0000	3.202	2.9844	1.1388
-0.5	3.650	0.00	1.12	3.0000	2.036	4.5261	1.1256
-0.5	3.650	0.00	1.12	4.0000	1.505	6.0222	1.1153
-0.7	3.780	0.00	1.22	0.0000	373.134	-0.0022	1.2224
-0.7	3.780	0.00	1.22	0.5000	41.516	0.8459	1.2248
-0.7	3.780	0.00	1.22	1.0000	9.635	1.7070	1.2344
-0.7	3.780	0.00	1.22	2.0000	3.176	3.3809	1.2450
-0.7	3.780	0.00	1.22	3.0000	2.029	5.1319	1.2243
-0.7	3.780	0.00	1.22	4.0000	1.490	6.8198	1.2162

TABLE 8

Residuals of AR(1) Model Transformed to an MA(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.97**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAX=0.0027

LAMBDA= 0.25 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	1.585	0.00	1.35	0.0000	370.370	-0.0002	1.3457
0.9	1.585	0.00	1.35	0.5000	370.370	0.0048	1.3457
0.9	1.585	0.00	1.35	1.0000	373.134	0.0098	1.3457
0.9	1.585	0.00	1.35	2.0000	367.647	0.0198	1.3457
0.9	1.585	0.00	1.35	3.0000	342.466	0.0298	1.3457
0.9	1.585	0.00	1.35	4.0000	328.947	0.0398	1.3457
0.7	1.680	0.00	1.22	0.0000	370.370	-0.0004	1.2210
0.7	1.680	0.00	1.22	0.5000	316.456	0.0446	1.2210
0.7	1.680	0.00	1.22	1.0000	221.239	0.0896	1.2210
0.7	1.680	0.00	1.22	2.0000	102.881	0.1796	1.2210
0.7	1.680	0.00	1.22	3.0000	50.968	0.2696	1.2210
0.7	1.680	0.00	1.22	4.0000	27.631	0.3596	1.2240
0.5	1.910	0.00	1.12	0.0000	364.964	-0.0007	1.1185
0.5	1.910	0.00	1.12	0.5000	186.567	0.1243	1.1185
0.5	1.910	0.00	1.12	1.0000	68.306	0.2493	1.1185
0.5	1.910	0.00	1.12	2.0000	17.491	0.4985	1.1206
0.5	1.910	0.00	1.12	3.0000	8.403	0.7546	1.1348
0.5	1.910	0.00	1.12	4.0000	5.658	1.0029	1.1407
0.3	2.290	0.00	1.04	0.0000	373.134	-0.0009	1.0446
0.3	2.290	0.00	1.04	0.5000	94.697	0.2441	1.0446
0.3	2.290	0.00	1.04	1.0000	25.395	0.4866	1.0469
0.3	2.290	0.00	1.04	2.0000	6.950	0.9902	1.0631
0.3	2.290	0.00	1.04	3.0000	4.038	1.4672	1.0675
0.3	2.290	0.00	1.04	4.0000	2.836	1.9564	1.0781
0.1	2.725	0.00	1.01	0.0000	367.647	-0.0012	1.0057
0.1	2.725	0.00	1.01	0.5000	56.180	0.4038	1.0057
0.1	2.725	0.00	1.01	1.0000	13.239	0.8100	1.0079
0.1	2.725	0.00	1.01	2.0000	4.292	1.6193	1.0251
0.1	2.725	0.00	1.01	3.0000	2.583	2.4323	1.0348
0.1	2.725	0.00	1.01	4.0000	1.930	3.2658	1.0319

TABLE 8

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	3.160	0.00	1.01	0.0000	373.134	-0.0014	1.0060
-0.1	3.160	0.00	1.01	0.5000	36.004	0.6031	1.0100
-0.1	3.160	0.00	1.01	1.0000	8.617	1.2175	1.0206
-0.1	3.160	0.00	1.01	2.0000	3.067	2.4022	1.0279
-0.1	3.160	0.00	1.01	3.0000	1.986	3.6555	1.0238
-0.1	3.160	0.00	1.01	4.0000	1.478	4.8504	1.0121
-0.3	3.460	0.00	1.05	0.0000	367.647	-0.0017	1.0453
-0.3	3.460	0.00	1.05	0.5000	24.089	0.8367	1.0540
-0.3	3.460	0.00	1.05	1.0000	6.130	1.6991	1.0593
-0.3	3.460	0.00	1.05	2.0000	2.403	3.3935	1.0639
-0.3	3.460	0.00	1.05	3.0000	1.620	5.0868	1.0656
-0.3	3.460	0.00	1.05	4.0000	1.095	6.7569	1.0641
-0.5	3.650	0.00	1.12	0.0000	370.370	-0.0019	1.1195
-0.5	3.650	0.00	1.12	0.5000	17.039	1.1204	1.1290
-0.5	3.650	0.00	1.12	1.0000	4.772	2.2540	1.1302
-0.5	3.650	0.00	1.12	2.0000	2.038	4.5259	1.1251
-0.5	3.650	0.00	1.12	3.0000	1.257	6.7543	1.1230
-0.5	3.650	0.00	1.12	4.0000	1.006	8.9649	1.1171
-0.7	3.780	0.00	1.22	0.0000	373.134	-0.0022	1.2224
-0.7	3.780	0.00	1.22	0.5000	13.143	1.4465	1.2339
-0.7	3.780	0.00	1.22	1.0000	3.966	2.8837	1.2391
-0.7	3.780	0.00	1.22	2.0000	1.813	5.8231	1.2451
-0.7	3.780	0.00	1.22	3.0000	1.068	8.6604	1.2163
-0.7	3.780	0.00	1.22	4.0000	1.002	11.5202	1.2136

the z's and the distribution of the y's from an MA(1) process are compared to the distribution of the transformed x(t) individually to explain the results obtained. In the earlier approach we had compared the ARL of the z's and the ARL of the y's by comparing the ratio of their magnitude of the shift with respect to their control limits . So for a shift = 4.0 in the y's and θ = 0.7 we had seen that the average run length of the z's is smaller than the average run length of the y's. So it is enough to compare the ARL of the z's and the x's because it has already been shown that the ARL for the z's are better than that of the y's. The z's upon introduction of the shift in the process output or the y's have transformed from a NID(0,1) to a distribution whose mean = 13.32 and standard deviation = 1.00 [See Figure-5 & Table-2] . The x(t) have likewise been transformed from a (0,1.41) to a (44.39,1.41) distribution [Figure-18 & Table-6] .

Shift in x's / Shift in z's =(44.3/1.4)σ/(13.3/1.0)σ=2.37

Control Limit factor ratio = 2.97/1.705= 1.74

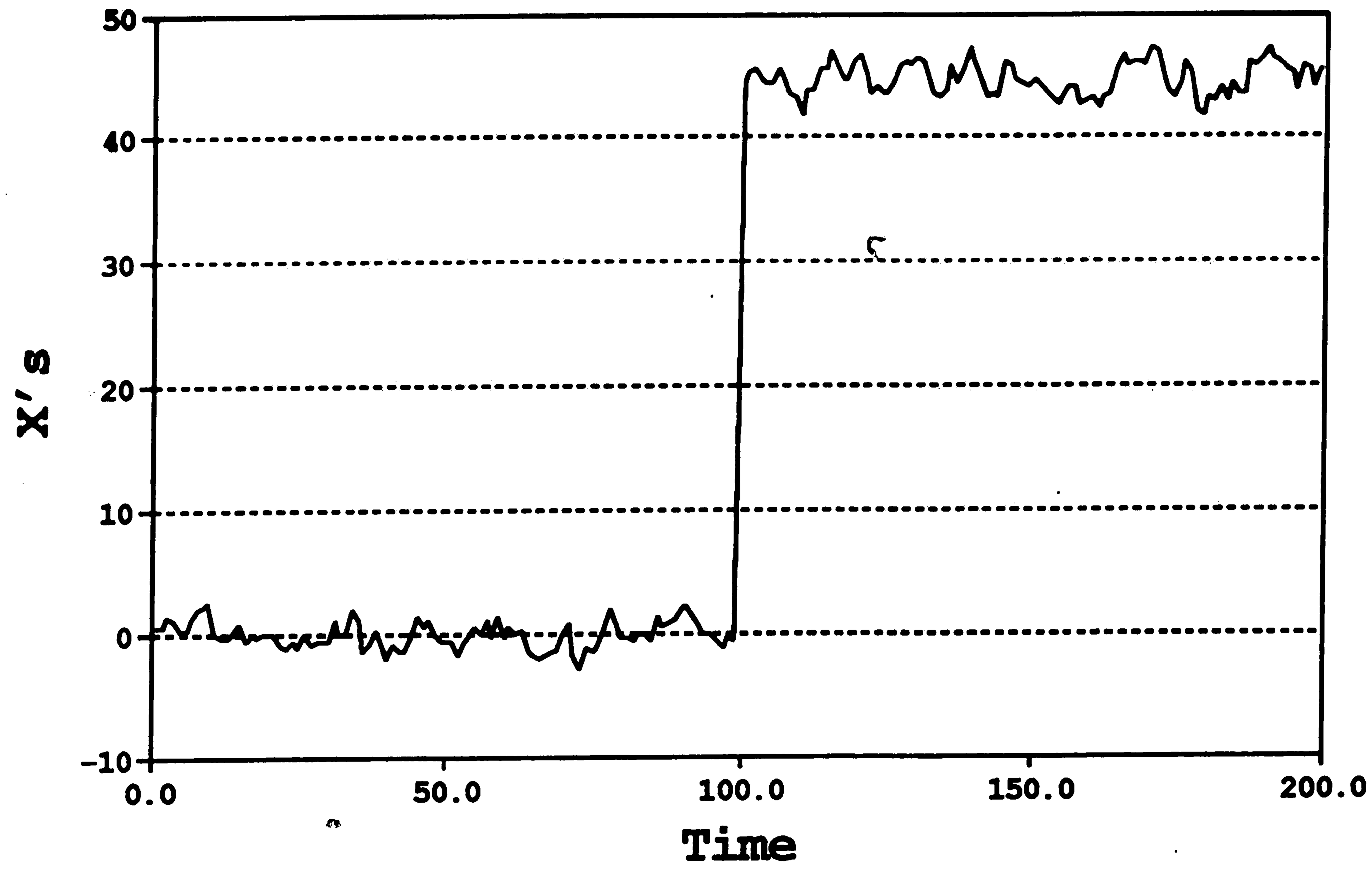
Since 2.37 > 1.74 the ARL of the x's is better. The same technique may be used to analyze results for different ARMA parameters[Figure-19 & Table-6] . Again this simplified analysis agrees with the simulation results. The transients induced in x(t) seems to have little effect.

4.1.3 AR(1) Model - Shift in mean due to input or uncorrelated shocks:

When the process model is AR(1) and the shift occurs at the input,the ARL's of x(t) [See Table-7] behaves exactly like the ARL's of y(t) for a MA(1) process [See Figure-2 & Table-1] and the results for the MA(1) model whose shift in mean was at the input apply

Figure - 18 MA(1) Model

Shift=4.0(Output) Lambda=0.25 THETA=0.7



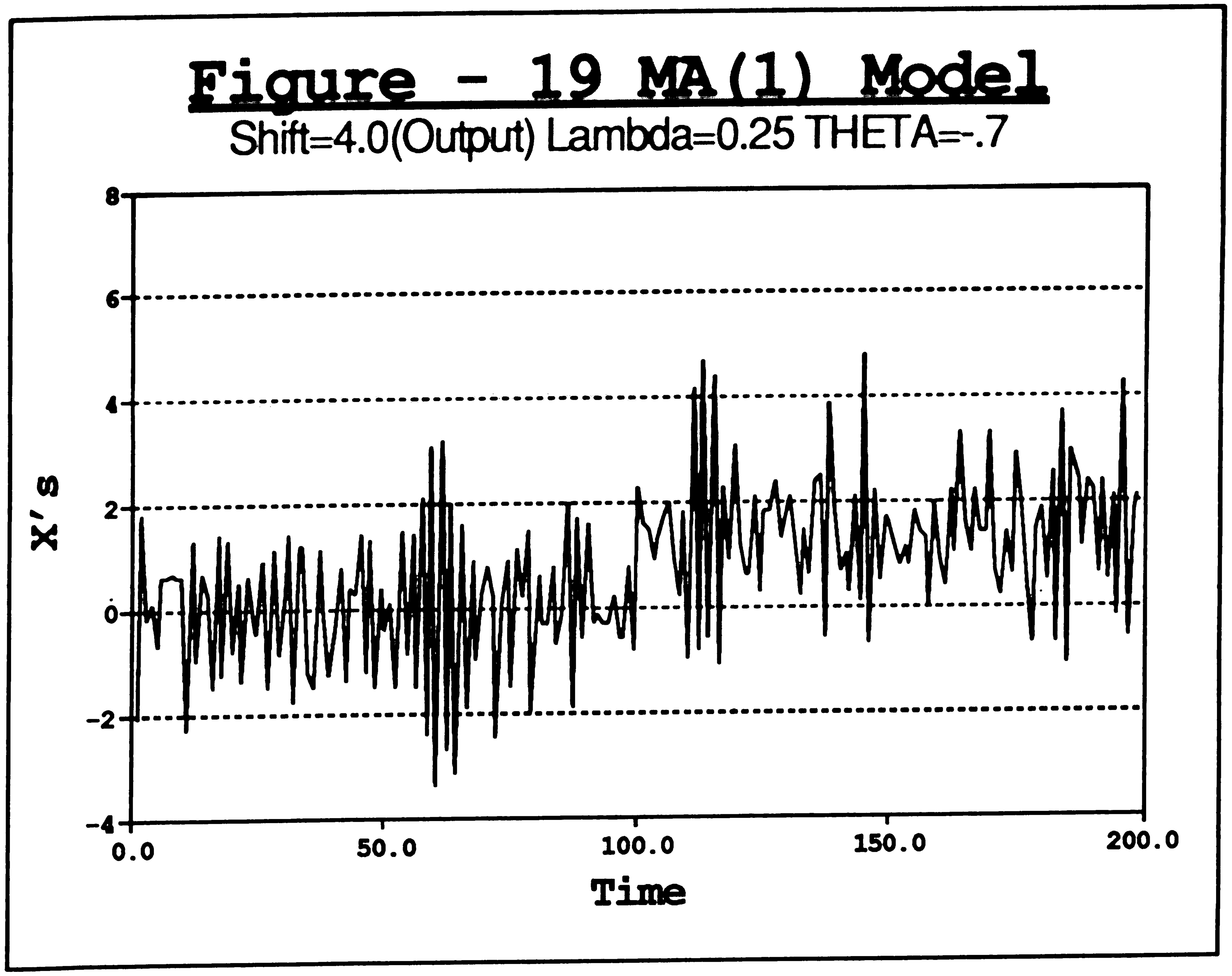


Figure - 19 MA(1) Model
Shift=4.0(Output) Lambda=0.25 THETA=-.7
8
6
4
2
0
-2
-4
X's
0.0
50.0
100.0
150.0
200.0
Time

here as well. In other words,

$$x(t) = z'(t) - \theta * z'(t-1) = y'(t) \text{ for a MA(1) process}$$

4.1.4 AR(1) Model - Shift in mean due to step change in process output:

Here the process model is AR(1) and the shift occurs at the output. The distributions of the z's and the distribution of the y's from an AR(1) process are compared to the distribution of the transformed x(t) individually to explain the results obtained. In the earlier approach we had compared the ARL of the z's and the ARL of the y's by comparing the ratio of their magnitude of the shift with respect to their control limits . So for a shift = 4.0 in the y's and $\theta = 0.7$ we had seen that the average run length of the y's is smaller than the average run length of the z's. So it is enough to compare the ARL of the y's and the x's . Upon introduction of the shift in the process output or the y's have transformed from a(0,1.41) to a distribution whose mean = 4.00 and standard deviation = 1.41[See Figure-13 & Table-4] . The x(t) have likewise been transformed from a (0,1.22) to a (0.36,1.22) distribution[See Figure-20 & Table-8] .

$$\text{Shift in y's / Shift in x's} =(4.0/1.4)\sigma/(0.36/1.22)\sigma= 11.65$$

$$\text{Control Limit factor ratio} = 4.88/1.68 = 2.90$$

Since 11.65>> 2.90 the ARL of the x's is better. The same technique may be used to analyze results for different ARMA parameters .[See Figure-21] .

Figure - 20 AR(1) Model

Shift=4.0(Output) Lambda=0.25 Phi=0.7

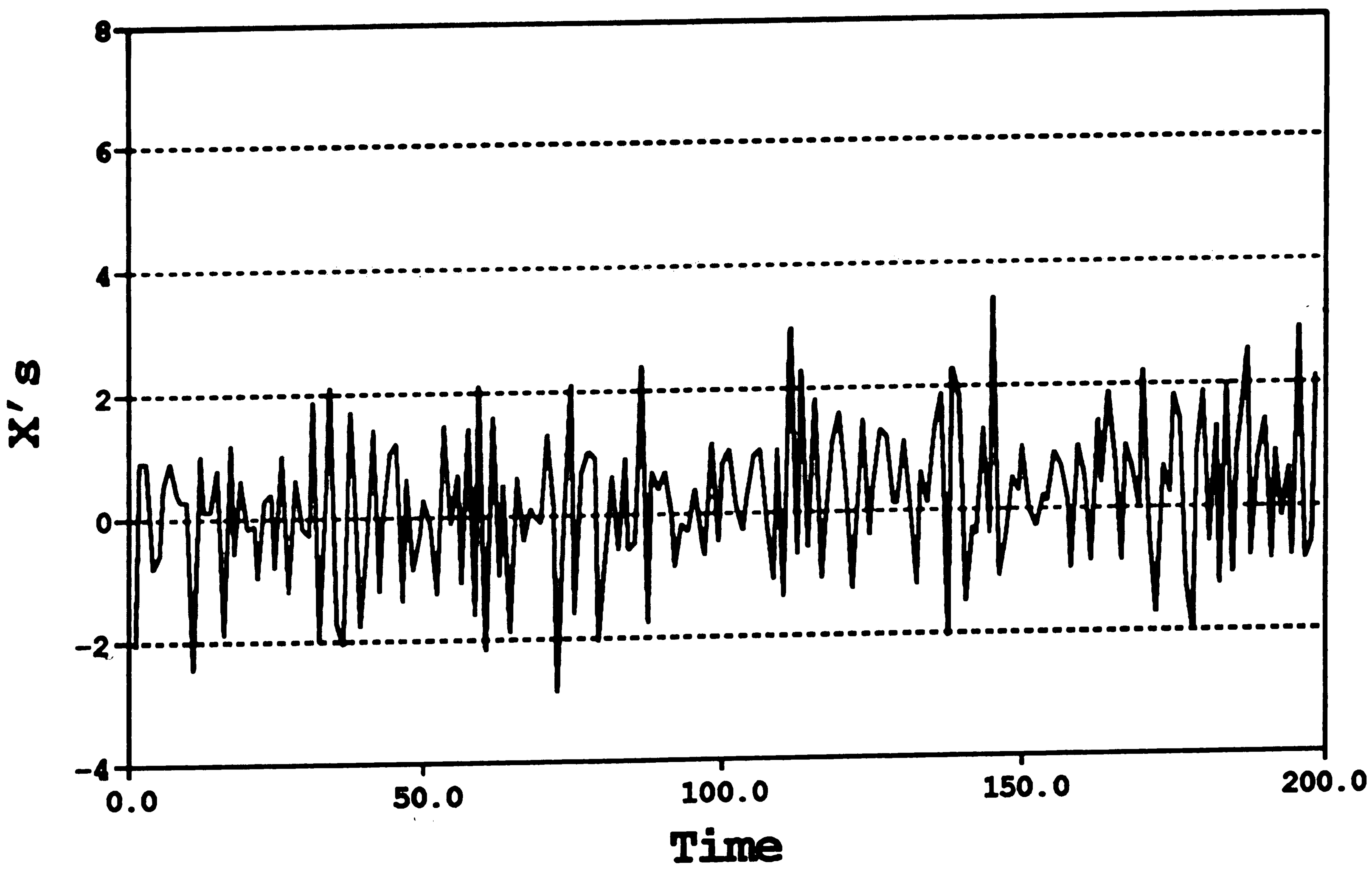
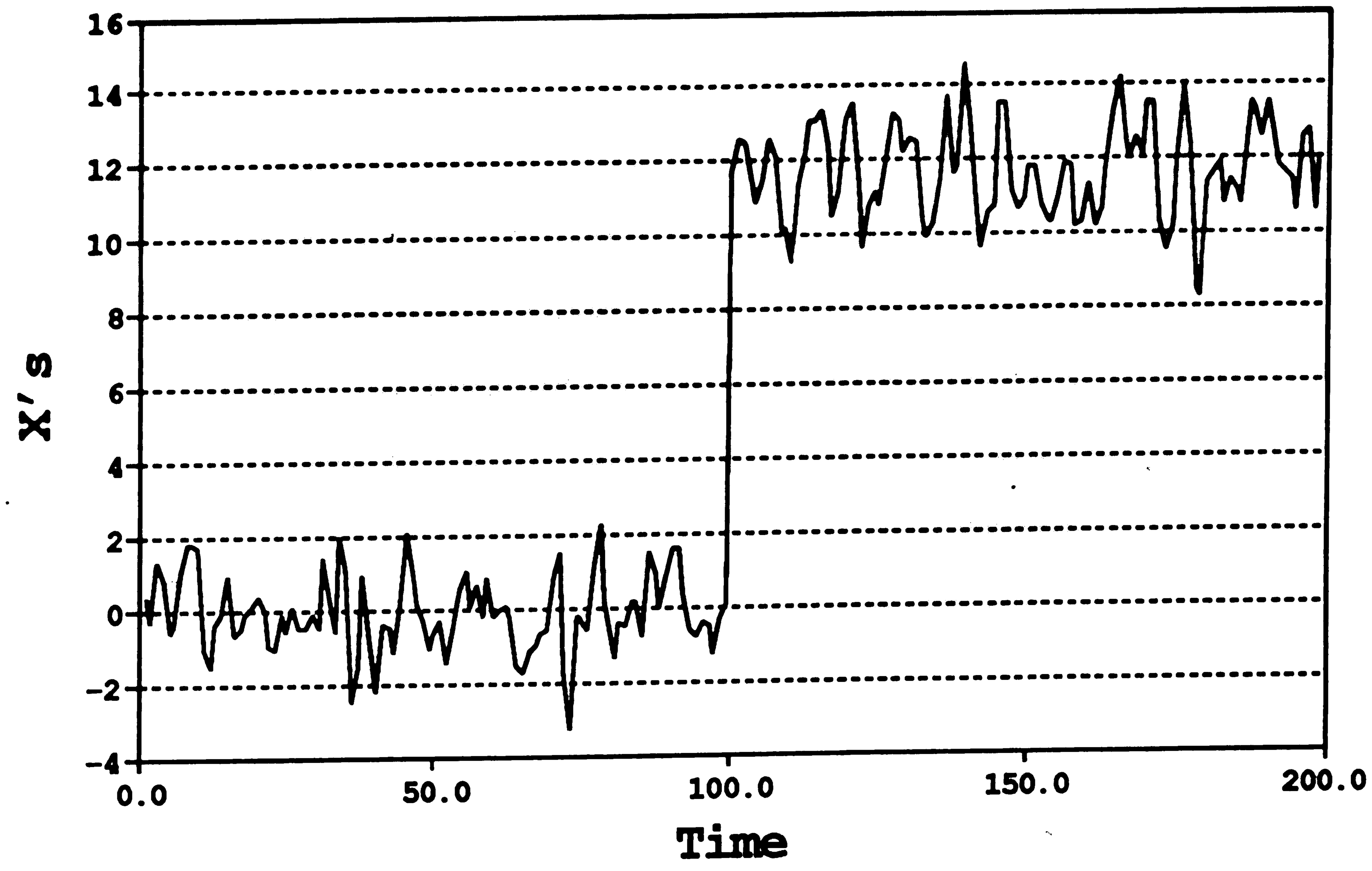


Figure - 21 AR(1) Model

Shift=4.0(Output) Lambda=0.25 PHI=-0.7



4.2.0 Effects of Transformation:

The effectiveness of using the second or alternate approach can be illustrated with the help of an example. If the process under consideration is an AR(1) process whose step change in mean is due to a step change in the input, then for ARMA parameter less than zero the ARL of y's obtained from an AR(1) process is not as good as either the ARL of z's or the ARL of the y's obtained from an MA(1) process. Under such a circumstance it might be better to monitor the process by using the z's to detect the shift in mean. A better approach would be to acquire the properties of the ARL of y's of an MA(1) process whose performance is superior even when compared to the performance of the ARL of z's. This can be done by transforming the residuals of an AR(1) process to an MA(1) process. So even though the actual manufacturing process is AR(1), transformation of the residuals has led to the model acquiring the properties of an MA(1) model. See Figure - 22 for a graphic illustration of the transformation.

Likewise if the actual manufacturing process was an MA(1) model and if its ARMA parameter was greater than zero then the opposite tranformation to an AR(1) model would yield good results.See Figure - 23 for a graphic illustration of the transformation.

The above examples were for the transformation iin the face of a shift in the mean of the process due to a step change in the mean of the input. The same has been found to be true for a model whose shift in the mean was due to a step change in the mean of the process output.

So depending upon the nature of the basic process and the value of its ARMA parameter suitable transformation, if necessary, can be made to achieve better control of the process.

Figure - 22 AR(1) Transformed to MA(1)

Shift = 2.0 (Input) Lambda = 0.50

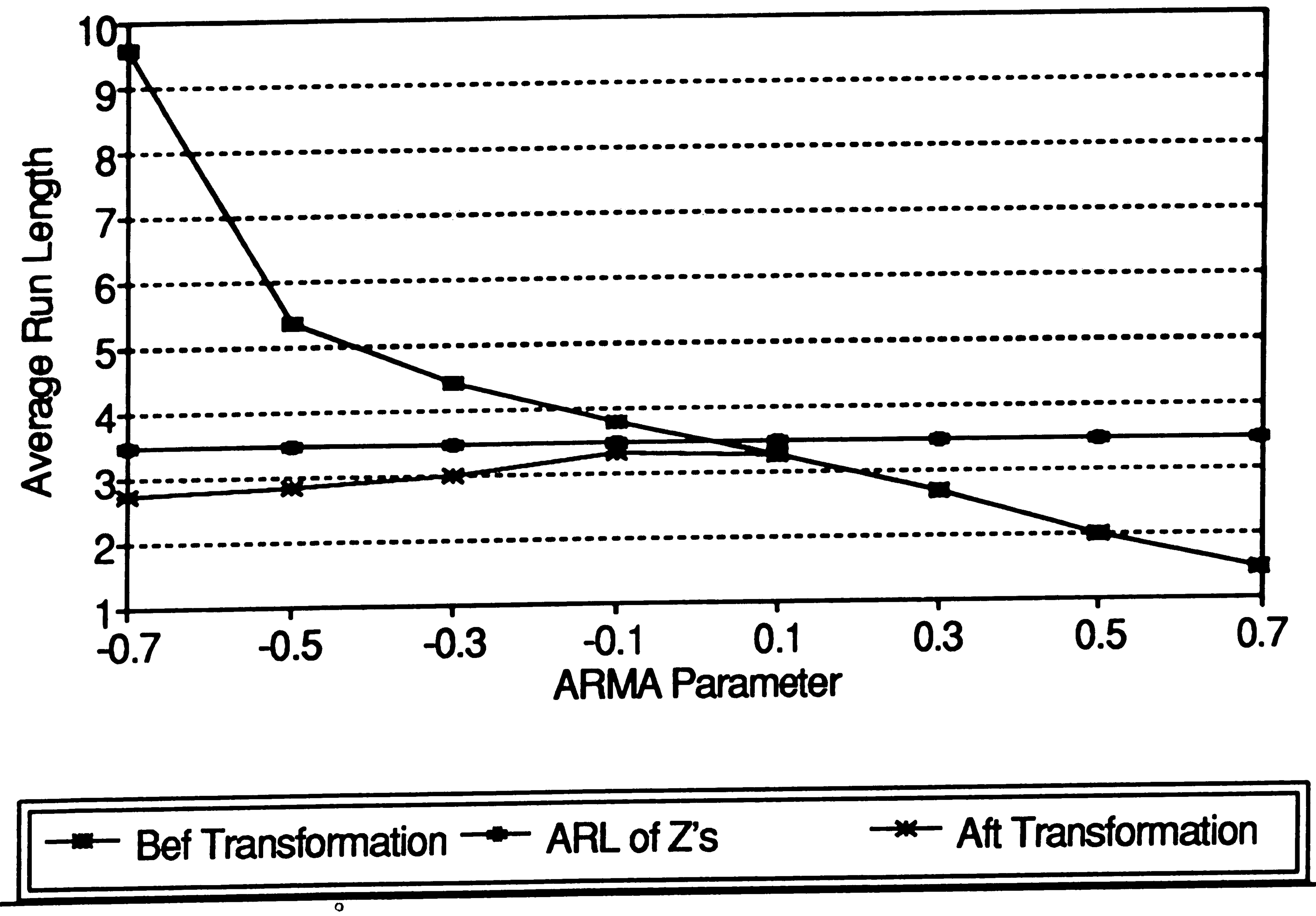
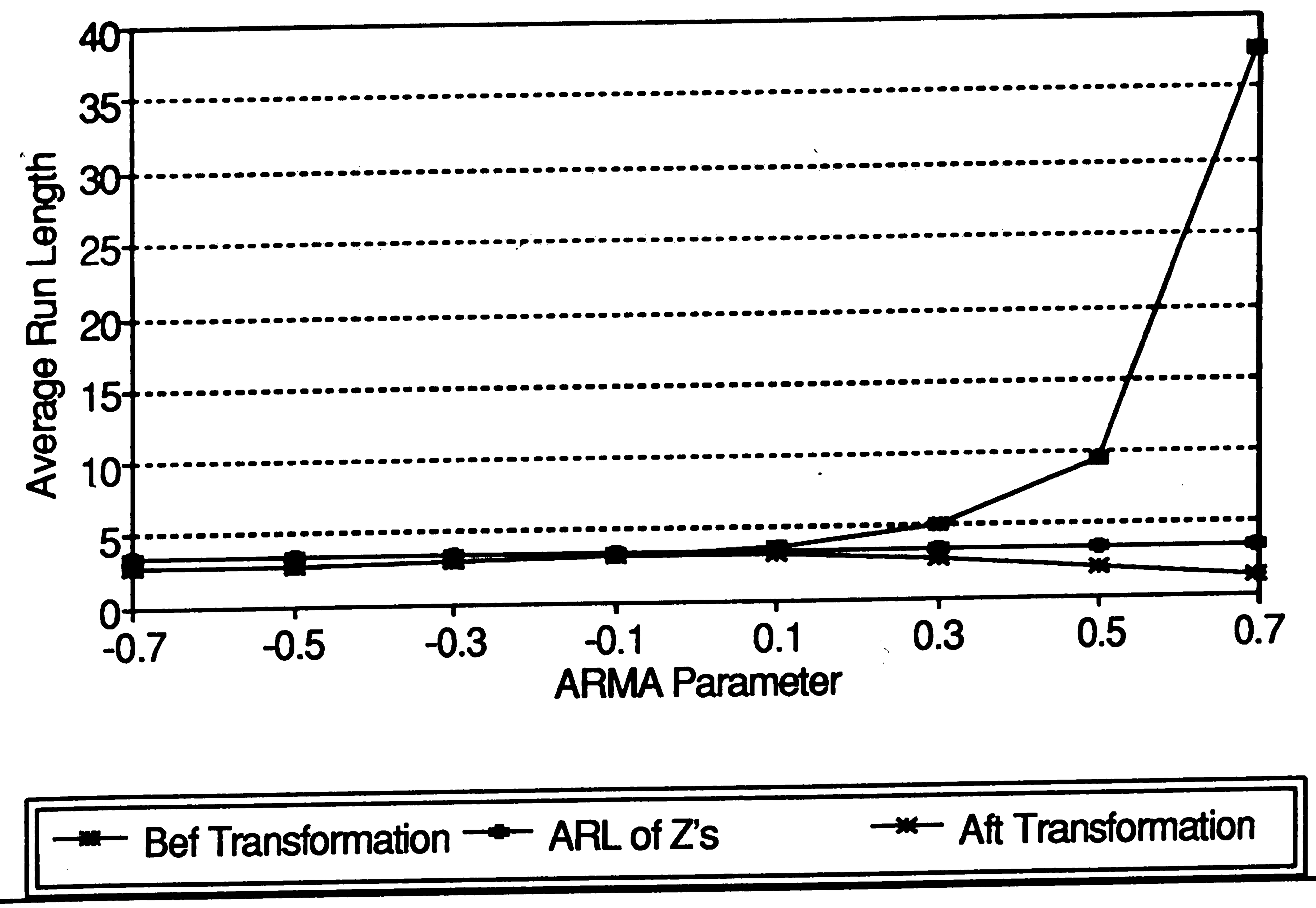


Figure - 23 MA(1) Transformed to AR(1)
Shift = 2.0 (Input) Lambda = 0.50



5.0 Conclusions:

In the ultimate analysis we would like to conclude by making several recommendations for monitoring a process whose output is serially correlated. By using the two approaches we have proposed better control of the manufacturing processes can be achieved.

The first approach we have proposed works well under certain circumstances while the second approach works better under a different set of circumstances. Depending upon the process under consideration, nature of the process, nature of the shift in the mean of the process and the value of the ARMA parameter different control strategies can be suggested.

For each time series model we can envision two different environments. This is with respect to the shift in the mean of the process. The shift in the mean can either be due to the input of the process or the output of the process. Depending upon the nature of the shift different approaches are required. The same is true of the ARMA parameter. The models behave differently under different values of the ARMA parameter. So a knowledge of these factors can lead to the determination of the type of approach to be used for the purposes of control ling a manufacturing process whose output is serially correlated. This would also mean the choice of control limits for control charts. Different values of control limits under different environments have to be used and the control limit factors have been arrived at for different environments. These are presented in Tables 1- 24. Given below is a broad set of guidelines to be used for monitoring correlated processes.

Process Model	Type of Shift	Recommended Action
MA(1)	Input	If $\theta > 0$, transform z's to an AR(1) model & plot x(t) on a GMA chart If $\theta < 0$, plot y's on GMA chart. Transformation is not required.
AR(1)	Input	If $\phi < 0$, transform z's to an MA(1) model & plot x(t) on GMA chart. If $\phi > 0$, plot y's on GMA chart. Transformation is not required.
MA(1)	Output	If $\theta > 0$, transform z's to an AR(1) model & plot x(t) on GMA chart If $\theta < 0$, plot y's on GMA chart. Transformation is not required.
AR(1)	Output	If $\phi < 0$, transform z's to an

MA(1) model & plot x(t) on

GMA chart.

If $\phi>0$, plot y's on GMA

chart. Transformation is not

required.

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APPENDIX A

TABLE 9

MA(1) Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	2.215	0.00	0.00	1.35	1.00	0.0	362.32	362.32	0.00	1.35	0.00	1.00
0.9	2.215	0.00	0.00	1.35	1.00	0.5	359.71	74.40	0.05	1.35	0.50	1.00
0.9	2.215	0.00	0.00	1.35	1.00	1.0	340.14	15.10	0.10	1.35	1.00	1.00
0.9	2.215	0.00	0.00	1.35	1.00	2.0	279.33	3.48	0.20	1.35	1.99	1.02
0.9	2.215	0.00	0.00	1.35	1.00	3.0	181.82	1.85	0.30	1.35	3.02	1.03
0.9	2.215	0.00	0.00	1.35	1.00	4.0	109.41	1.30	0.40	1.35	4.00	1.01
0.7	2.250	0.00	0.00	1.22	1.00	0.0	373.13	362.32	0.00	1.22	0.00	1.00
0.7	2.250	0.00	0.00	1.22	1.00	0.5	289.02	74.40	0.15	1.22	0.50	1.00
0.7	2.250	0.00	0.00	1.22	1.00	1.0	155.76	15.10	0.30	1.22	1.00	1.00
0.7	2.250	0.00	0.00	1.22	1.00	2.0	37.75	3.48	0.60	1.23	1.99	1.02
0.7	2.250	0.00	0.00	1.22	1.00	3.0	13.83	1.85	0.90	1.22	3.02	1.03
0.7	2.250	0.00	0.00	1.22	1.00	4.0	6.67	1.30	1.20	1.24	4.00	1.01
0.5	2.350	0.00	0.00	1.12	1.00	0.0	370.37	362.32	0.00	1.12	0.00	1.00
0.5	2.350	0.00	0.00	1.12	1.00	0.5	162.87	74.40	0.25	1.12	0.50	1.00
0.5	2.350	0.00	0.00	1.12	1.00	1.0	50.81	15.10	0.50	1.12	1.00	1.00
0.5	2.350	0.00	0.00	1.12	1.00	2.0	9.44	3.48	1.00	1.13	1.99	1.02
0.5	2.350	0.00	0.00	1.12	1.00	3.0	4.21	1.85	1.50	1.15	3.02	1.03
0.5	2.350	0.00	0.00	1.12	1.00	4.0	2.76	1.30	2.00	1.16	4.00	1.01
0.3	2.550	0.00	0.00	1.04	1.00	0.0	370.37	362.32	0.00	1.04	0.00	1.00
0.3	2.550	0.00	0.00	1.04	1.00	0.5	101.01	74.40	0.35	1.04	0.50	1.00
0.3	2.550	0.00	0.00	1.04	1.00	1.0	24.11	15.10	0.70	1.05	1.00	1.00
0.3	2.550	0.00	0.00	1.04	1.00	2.0	5.07	3.48	1.40	1.06	1.99	1.02
0.3	2.550	0.00	0.00	1.04	1.00	3.0	2.56	1.85	2.10	1.08	3.02	1.03
0.3	2.550	0.00	0.00	1.04	1.00	4.0	1.79	1.30	2.82	1.08	4.00	1.01
0.1	2.850	0.00	0.00	1.01	1.00	0.0	362.32	362.32	0.00	1.01	0.00	1.00
0.1	2.850	0.00	0.00	1.01	1.00	0.5	81.17	74.40	0.45	1.01	0.50	1.00
0.1	2.850	0.00	0.00	1.01	1.00	1.0	16.67	15.10	0.90	1.01	1.00	1.00
0.1	2.850	0.00	0.00	1.01	1.00	2.0	3.80	3.48	1.80	1.03	1.99	1.02
0.1	2.850	0.00	0.00	1.01	1.00	3.0	2.00	1.85	2.72	1.03	3.02	1.03
0.1	2.850	0.00	0.00	1.01	1.00	4.0	1.44	1.30	3.60	1.01	4.00	1.01

TABLE 9

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	3.145	0.00	0.00	1.01	1.00	0.0	373.13	362.32	0.00	1.01	0.00	1.00
-0.1	3.145	0.00	0.00	1.01	1.00	0.5	72.78	74.40	0.55	1.01	0.50	1.00
-0.1	3.145	0.00	0.00	1.01	1.00	1.0	14.33	15.10	1.10	1.01	1.00	1.00
-0.1	3.145	0.00	0.00	1.01	1.00	2.0	3.29	3.48	2.19	1.02	1.99	1.02
-0.1	3.145	0.00	0.00	1.01	1.00	3.0	1.75	1.85	3.33	1.04	3.02	1.03
-0.1	3.145	0.00	0.00	1.01	1.00	4.0	1.23	1.30	4.40	1.01	4.00	1.01
-0.3	3.380	0.00	0.00	1.05	1.00	0.0	373.13	362.32	0.00	1.05	0.00	1.00
-0.3	3.380	0.00	0.00	1.05	1.00	0.5	70.32	74.40	0.65	1.05	0.50	1.00
-0.3	3.380	0.00	0.00	1.05	1.00	1.0	13.10	15.10	1.30	1.05	1.00	1.00
-0.3	3.380	0.00	0.00	1.05	1.00	2.0	3.02	3.48	2.59	1.07	1.99	1.02
-0.3	3.380	0.00	0.00	1.05	1.00	3.0	1.59	1.85	3.92	1.07	3.02	1.03
-0.3	3.380	0.00	0.00	1.05	1.00	4.0	1.12	1.30	5.19	1.06	4.00	1.01
-0.5	3.520	0.00	0.00	1.12	1.00	0.0	362.32	362.32	0.00	1.12	0.00	1.00
-0.5	3.520	0.00	0.00	1.12	1.00	0.5	66.76	74.40	0.75	1.12	0.50	1.00
-0.5	3.520	0.00	0.00	1.12	1.00	1.0	12.47	15.10	1.50	1.13	1.00	1.00
-0.5	3.520	0.00	0.00	1.12	1.00	2.0	2.83	3.48	2.99	1.15	1.99	1.02
-0.5	3.520	0.00	0.00	1.12	1.00	3.0	1.51	1.85	4.52	1.11	3.02	1.03
-0.5	3.520	0.00	0.00	1.12	1.00	4.0	1.09	1.30	5.99	1.14	4.00	1.01
-0.7	3.580	0.00	0.00	1.22	1.00	0.0	364.96	362.32	0.00	1.22	0.00	1.00
-0.7	3.580	0.00	0.00	1.22	1.00	0.5	63.05	74.40	0.85	1.22	0.50	1.00
-0.7	3.580	0.00	0.00	1.22	1.00	1.0	11.89	15.10	1.70	1.24	1.00	1.00
-0.7	3.580	0.00	0.00	1.22	1.00	2.0	2.71	3.48	3.40	1.25	1.99	1.02
-0.7	3.580	0.00	0.00	1.22	1.00	3.0	1.46	1.84	5.11	1.21	3.02	1.03
-0.7	3.580	0.00	0.00	1.22	1.00	4.0	1.06	1.30	6.79	1.22	4.00	1.01

TABLE 10

MA(1) Model
********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********
***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT
ALPHAY=ALPHAZ=0.0027
LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	2.215	0.00	0.00	1.35	1.00	0.0	362.32	362.32	0.00	1.35	0.00	1.00
0.9	2.215	0.00	0.00	1.35	1.00	0.5	73.31	1.07	0.50	1.35	4.99	1.01
0.9	2.215	0.00	0.00	1.35	1.00	1.0	12.95	1.00	1.00	1.34	9.98	1.01
0.9	2.215	0.00	0.00	1.35	1.00	2.0	3.23	1.00	2.00	1.38	19.99	1.01
0.9	2.215	0.00	0.00	1.35	1.00	3.0	1.93	1.00	3.00	1.40	30.00	1.01
0.9	2.215	0.00	0.00	1.35	1.00	4.0	1.43	1.00	4.00	1.37	40.01	1.01
0.7	2.250	0.00	0.00	1.22	1.00	0.0	373.13	362.32	0.00	1.22	0.00	1.00
0.7	2.250	0.00	0.00	1.22	1.00	0.5	58.34	4.81	0.50	1.22	1.67	1.02
0.7	2.250	0.00	0.00	1.22	1.00	1.0	10.41	1.64	1.00	1.23	3.35	1.03
0.7	2.250	0.00	0.00	1.22	1.00	2.0	2.89	1.00	2.00	1.26	6.64	1.01
0.7	2.250	0.00	0.00	1.22	1.00	3.0	1.74	1.00	3.01	1.27	9.98	1.01
0.7	2.250	0.00	0.00	1.22	1.00	4.0	1.31	1.00	4.00	1.25	13.32	1.01
0.5	2.350	0.00	0.00	1.12	1.00	0.0	370.37	362.32	0.00	1.12	0.00	1.00
0.5	2.350	0.00	0.00	1.12	1.00	0.5	50.81	15.10	0.50	1.12	1.00	1.00
0.5	2.350	0.00	0.00	1.12	1.00	1.0	9.44	3.48	1.00	1.13	1.99	1.02
0.5	2.350	0.00	0.00	1.12	1.00	2.0	2.76	1.30	2.00	1.16	4.00	1.01
0.5	2.350	0.00	0.00	1.12	1.00	3.0	1.65	1.01	3.01	1.16	5.98	1.01
0.5	2.350	0.00	0.00	1.12	1.00	4.0	1.23	1.00	4.00	1.14	7.98	1.01
0.3	2.550	0.00	0.00	1.04	1.00	0.0	370.37	362.32	0.00	1.04	0.00	1.00
0.3	2.550	0.00	0.00	1.04	1.00	0.5	50.25	35.20	0.50	1.04	0.71	1.01
0.3	2.550	0.00	0.00	1.04	1.00	1.0	10.12	6.59	1.00	1.05	1.44	1.01
0.3	2.550	0.00	0.00	1.04	1.00	2.0	2.79	1.96	2.00	1.08	2.88	1.02
0.3	2.550	0.00	0.00	1.04	1.00	3.0	1.66	1.20	3.01	1.08	4.29	1.01
0.3	2.550	0.00	0.00	1.04	1.00	4.0	1.21	1.02	4.00	1.06	5.69	1.02
0.1	2.850	0.00	0.00	1.01	1.00	0.0	362.32	362.32	0.00	1.01	0.00	1.00
0.1	2.850	0.00	0.00	1.01	1.00	0.5	65.36	60.39	0.50	1.01	0.55	1.00
0.1	2.850	0.00	0.00	1.01	1.00	1.0	13.26	11.57	1.00	1.01	1.11	1.01
0.1	2.850	0.00	0.00	1.01	1.00	2.0	3.22	2.90	1.99	1.03	2.22	1.03
0.1	2.850	0.00	0.00	1.01	1.00	3.0	1.76	1.64	3.02	1.04	3.35	1.03
0.1	2.850	0.00	0.00	1.01	1.00	4.0	1.26	1.15	4.00	1.02	4.45	1.02

TABLE 10

THET	Contrl Limit	Av.Y **BEFORE SHIFT HAS OCCURED**	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y ***AFTER THE SHIFT HAS OCCURED***	Av.Z	Sd.Y	Sd.Z
-0.1	3.145	0.00	0.00	1.01	1.00	0.0	373.13	362.32	0.00	1.01	0.00	1.00
-0.1	3.145	0.00	0.00	1.01	1.00	0.5	86.51	86.81	0.50	1.01	0.45	1.00
-0.1	3.145	0.00	0.00	1.01	1.00	1.0	18.30	18.99	0.99	1.01	0.90	1.01
-0.1	3.145	0.00	0.00	1.01	1.00	2.0	3.87	4.13	2.00	1.03	1.81	1.02
-0.1	3.145	0.00	0.00	1.01	1.00	3.0	1.98	2.08	3.03	1.02	2.74	1.02
-0.1	3.145	0.00	0.00	1.01	1.00	4.0	1.36	1.47	4.01	1.02	3.65	1.01
-0.3	3.380	0.00	0.00	1.05	1.00	0.0	373.13	362.32	0.00	1.05	0.00	1.00
-0.3	3.380	0.00	0.00	1.05	1.00	0.5	107.99	114.94	0.50	1.05	0.38	1.00
-0.3	3.380	0.00	0.00	1.05	1.00	1.0	25.67	27.78	0.99	1.05	0.77	1.01
-0.3	3.380	0.00	0.00	1.05	1.00	2.0	4.90	5.67	2.01	1.06	1.54	1.02
-0.3	3.380	0.00	0.00	1.05	1.00	3.0	2.33	2.71	3.00	1.06	2.31	1.03
-0.3	3.380	0.00	0.00	1.05	1.00	4.0	1.54	1.78	4.02	1.06	3.10	1.03
-0.5	3.520	0.00	0.00	1.12	1.00	0.0	362.32	362.32	0.00	1.12	0.00	1.00
-0.5	3.520	0.00	0.00	1.12	1.00	0.5	132.63	138.12	0.50	1.12	0.33	1.00
-0.5	3.520	0.00	0.00	1.12	1.00	1.0	34.44	41.12	1.00	1.13	0.66	1.00
-0.5	3.520	0.00	0.00	1.12	1.00	2.0	6.30	7.61	2.01	1.13	1.35	1.02
-0.5	3.520	0.00	0.00	1.12	1.00	3.0	2.83	3.48	2.99	1.15	1.99	1.02
-0.5	3.520	0.00	0.00	1.12	1.00	4.0	1.76	2.14	4.04	1.14	2.68	1.02
-0.7	3.580	0.00	0.00	1.22	1.00	0.0	364.96	362.32	0.00	1.22	0.00	1.00
-0.7	3.580	0.00	0.00	1.22	1.00	0.5	146.20	157.23	0.50	1.22	0.29	1.00
-0.7	3.580	0.00	0.00	1.22	1.00	1.0	44.12	53.59	1.00	1.22	0.59	1.00
-0.7	3.580	0.00	0.00	1.22	1.00	2.0	8.06	10.23	2.02	1.24	1.18	1.01
-0.7	3.580	0.00	0.00	1.22	1.00	3.0	3.52	4.37	2.99	1.24	1.77	1.02
-0.7	3.580	0.00	0.00	1.22	1.00	4.0	2.03	2.63	4.03	1.22	2.35	1.03

TABLE 11

AR(1) Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	4.450	-.01	0.00	2.33	1.00	0.0	364.96	362.32	-0.01	2.33	0.00	1.00
0.9	4.450	-.01	0.00	2.33	1.00	0.5	7.86	74.40	5.13	2.38	0.50	1.00
0.9	4.450	-.01	0.00	2.33	1.00	1.0	1.81	15.10	10.23	2.18	1.00	1.00
0.9	4.450	-.01	0.00	2.33	1.00	2.0	1.00	3.48	19.75	2.12	1.99	1.02
0.9	4.450	-.01	0.00	2.33	1.00	3.0	1.00	1.85	29.76	2.12	3.02	1.03
0.9	4.450	-.01	0.00	2.33	1.00	4.0	1.00	1.30	39.77	2.12	4.00	1.01
0.7	4.130	0.00	0.00	1.41	1.00	0.0	370.37	362.32	0.00	1.41	0.00	1.00
0.7	4.130	0.00	0.00	1.41	1.00	0.5	30.87	74.40	1.66	1.42	0.50	1.00
0.7	4.130	0.00	0.00	1.41	1.00	1.0	5.20	15.10	3.36	1.42	1.00	1.00
0.7	4.130	0.00	0.00	1.41	1.00	2.0	1.48	3.48	6.69	1.38	1.99	1.02
0.7	4.130	0.00	0.00	1.41	1.00	3.0	1.00	1.85	9.91	1.37	3.02	1.03
0.7	4.130	0.00	0.00	1.41	1.00	4.0	1.00	1.30	13.25	1.37	4.00	1.01
0.5	3.750	0.00	0.00	1.16	1.00	0.0	373.13	362.32	0.00	1.16	0.00	1.00
0.5	3.750	0.00	0.00	1.16	1.00	0.5	46.31	74.40	1.00	1.16	0.50	1.00
0.5	3.750	0.00	0.00	1.16	1.00	1.0	8.19	15.10	2.02	1.18	1.00	1.00
0.5	3.750	0.00	0.00	1.16	1.00	2.0	2.02	3.48	4.04	1.15	1.99	1.02
0.5	3.750	0.00	0.00	1.16	1.00	3.0	1.16	1.85	6.00	1.17	3.02	1.03
0.5	3.750	0.00	0.00	1.16	1.00	4.0	1.00	1.30	7.95	1.14	4.00	1.01
0.3	3.480	0.00	0.00	1.05	1.00	0.0	373.13	362.32	0.00	1.05	0.00	1.00
0.3	3.480	0.00	0.00	1.05	1.00	0.5	63.86	74.40	0.71	1.05	0.50	1.00
0.3	3.480	0.00	0.00	1.05	1.00	1.0	11.57	15.10	1.43	1.06	1.00	1.00
0.3	3.480	0.00	0.00	1.05	1.00	2.0	2.67	3.48	2.86	1.07	1.99	1.02
0.3	3.480	0.00	0.00	1.05	1.00	3.0	1.47	1.85	4.30	1.05	3.02	1.03
0.3	3.480	0.00	0.00	1.05	1.00	4.0	1.07	1.30	5.70	1.05	4.00	1.01
0.1	3.150	0.00	0.00	1.01	1.00	0.0	367.65	362.32	0.00	1.01	0.00	1.00
0.1	3.150	0.00	0.00	1.01	1.00	0.5	71.33	74.40	0.55	1.01	0.50	1.00
0.1	3.150	0.00	0.00	1.01	1.00	1.0	14.13	15.10	1.11	1.01	1.00	1.00
0.1	3.150	0.00	0.00	1.01	1.00	2.0	3.25	3.48	2.21	1.03	1.99	1.02
0.1	3.150	0.00	0.00	1.01	1.00	3.0	1.73	1.85	3.36	1.04	3.02	1.03
0.1	3.150	0.00	0.00	1.01	1.00	4.0	1.22	1.30	4.45	1.02	4.00	1.01

TABLE 11

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	2.860	0.00	0.00	1.01	1.00	0.0	367.65	362.32	0.00	1.01	0.00	1.00
-0.1	2.860	0.00	0.00	1.01	1.00	0.5	81.57	74.40	0.45	1.01	0.50	1.00
-0.1	2.860	0.00	0.00	1.01	1.00	1.0	16.45	15.10	0.91	1.01	1.00	1.00
-0.1	2.860	0.00	0.00	1.01	1.00	2.0	3.76	3.48	1.82	1.03	1.99	1.02
-0.1	2.860	0.00	0.00	1.01	1.00	3.0	2.00	1.85	2.75	1.03	3.02	1.03
-0.1	2.860	0.00	0.00	1.01	1.00	4.0	1.42	1.30	3.64	1.02	4.00	1.01
-0.3	2.580	0.00	0.00	1.05	1.00	0.0	362.32	362.32	0.00	1.05	0.00	1.00
-0.3	2.580	0.00	0.00	1.05	1.00	0.5	92.25	74.40	0.38	1.05	0.50	1.00
-0.3	2.580	0.00	0.00	1.05	1.00	1.0	20.46	15.10	0.77	1.05	1.00	1.00
-0.3	2.580	0.00	0.00	1.05	1.00	2.0	4.42	3.48	1.54	1.07	1.99	1.02
-0.3	2.580	0.00	0.00	1.05	1.00	3.0	2.33	1.85	2.31	1.09	3.02	1.03
-0.3	2.580	0.00	0.00	1.05	1.00	4.0	1.65	1.30	3.09	1.09	4.00	1.01
-0.5	2.320	0.00	0.00	1.16	1.00	0.0	364.96	362.32	0.00	1.16	0.00	1.00
-0.5	2.320	0.00	0.00	1.16	1.00	0.5	105.71	74.40	0.33	1.16	0.50	1.00
-0.5	2.320	0.00	0.00	1.16	1.00	1.0	27.35	15.10	0.67	1.16	1.00	1.00
-0.5	2.320	0.00	0.00	1.16	1.00	2.0	5.37	3.48	1.34	1.18	1.99	1.02
-0.5	2.320	0.00	0.00	1.16	1.00	3.0	2.76	1.85	2.00	1.20	3.02	1.03
-0.5	2.320	0.00	0.00	1.16	1.00	4.0	1.93	1.30	2.69	1.21	4.00	1.01
-0.7	2.160	0.00	0.00	1.41	1.00	0.0	364.96	362.32	0.00	1.41	0.00	1.00
-0.7	2.160	0.00	0.00	1.41	1.00	0.5	171.82	74.40	0.29	1.41	0.50	1.00
-0.7	2.160	0.00	0.00	1.41	1.00	1.0	57.34	15.10	0.59	1.41	1.00	1.00
-0.7	2.160	0.00	0.00	1.41	1.00	2.0	9.60	3.48	1.18	1.42	1.99	1.02
-0.7	2.160	0.00	0.00	1.41	1.00	3.0	3.92	1.85	1.76	1.44	3.02	1.03
-0.7	2.160	0.00	0.00	1.41	1.00	4.0	2.52	1.30	2.35	1.48	4.00	1.01

TABLE 12

AR(1) Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	4.450	-.01	0.00	2.33	1.00	0.0	364.96	362.32	-0.01	2.33	0.00	1.00
0.9	4.450	-.01	0.00	2.33	1.00	0.5	337.84	331.13	0.49	2.33	0.05	1.00
0.9	4.450	-.01	0.00	2.33	1.00	1.0	214.59	297.62	0.99	2.33	0.10	1.00
0.9	4.450	-.01	0.00	2.33	1.00	2.0	80.26	223.21	1.99	2.33	0.20	1.00
0.9	4.450	-.01	0.00	2.33	1.00	3.0	31.64	152.91	2.98	2.35	0.30	1.00
0.9	4.450	-.01	0.00	2.33	1.00	4.0	15.23	108.93	3.97	2.37	0.40	1.00
0.7	4.130	0.00	0.00	1.41	1.00	0.0	370.37	362.32	0.00	1.41	0.00	1.00
0.7	4.130	0.00	0.00	1.41	1.00	0.5	229.36	264.55	0.50	1.41	0.15	1.00
0.7	4.130	0.00	0.00	1.41	1.00	1.0	91.24	152.91	1.00	1.41	0.30	1.00
0.7	4.130	0.00	0.00	1.41	1.00	2.0	19.40	51.12	1.98	1.43	0.60	1.00
0.7	4.130	0.00	0.00	1.41	1.00	3.0	6.64	19.41	3.03	1.42	0.90	1.01
0.7	4.130	0.00	0.00	1.41	1.00	4.0	3.56	9.81	3.99	1.42	1.20	1.01
0.5	3.750	0.00	0.00	1.16	1.00	0.0	373.13	362.32	0.00	1.16	0.00	1.00
0.5	3.750	0.00	0.00	1.16	1.00	0.5	146.63	186.57	0.50	1.16	0.25	1.00
0.5	3.750	0.00	0.00	1.16	1.00	1.0	46.31	74.40	1.00	1.16	0.50	1.00
0.5	3.750	0.00	0.00	1.16	1.00	2.0	8.19	15.10	2.02	1.18	1.00	1.00
0.5	3.750	0.00	0.00	1.16	1.00	3.0	3.43	6.01	2.99	1.17	1.51	1.02
0.5	3.750	0.00	0.00	1.16	1.00	4.0	2.02	3.48	4.04	1.15	1.99	1.02
0.3	3.480	0.00	0.00	1.05	1.00	0.0	373.13	362.32	0.00	1.05	0.00	1.00
0.3	3.480	0.00	0.00	1.05	1.00	0.5	116.82	129.87	0.50	1.05	0.35	1.00
0.3	3.480	0.00	0.00	1.05	1.00	1.0	28.58	37.12	1.00	1.06	0.70	1.01
0.3	3.480	0.00	0.00	1.05	1.00	2.0	5.35	6.74	2.01	1.07	1.41	1.01
0.3	3.480	0.00	0.00	1.05	1.00	3.0	2.44	3.21	3.01	1.07	2.09	1.02
0.3	3.480	0.00	0.00	1.05	1.00	4.0	1.61	2.02	4.02	1.07	2.82	1.02
0.1	3.150	0.00	0.00	1.01	1.00	0.0	367.65	362.32	0.00	1.01	0.00	1.00
0.1	3.150	0.00	0.00	1.01	1.00	0.5	86.66	88.97	0.50	1.01	0.45	1.00
0.1	3.150	0.00	0.00	1.01	1.00	1.0	18.31	19.41	0.99	1.01	0.90	1.01
0.1	3.150	0.00	0.00	1.01	1.00	2.0	3.90	4.19	2.00	1.03	1.79	1.02
0.1	3.150	0.00	0.00	1.01	1.00	3.0	1.98	2.11	3.03	1.02	2.71	1.02
0.1	3.150	0.00	0.00	1.01	1.00	4.0	1.37	1.48	4.01	1.02	3.61	1.01

TABLE 12

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	2.860	0.00	0.00	1.01	1.00	0.0	367.65	362.32	0.00	1.01	0.00	1.00
-0.1	2.860	0.00	0.00	1.01	1.00	0.5	66.93	61.80	0.50	1.01	0.55	1.00
-0.1	2.860	0.00	0.00	1.01	1.00	1.0	13.37	11.86	1.00	1.01	1.10	1.01
-0.1	2.860	0.00	0.00	1.01	1.00	2.0	3.25	2.94	1.99	1.03	2.19	1.03
-0.1	2.860	0.00	0.00	1.01	1.00	3.0	1.77	1.65	3.02	1.04	3.31	1.03
-0.1	2.860	0.00	0.00	1.01	1.00	4.0	1.27	1.17	4.00	1.02	4.41	1.02
-0.3	2.580	0.00	0.00	1.05	1.00	0.0	362.32	362.32	0.00	1.05	0.00	1.00
-0.3	2.580	0.00	0.00	1.05	1.00	0.5	54.23	43.16	0.50	1.05	0.65	1.00
-0.3	2.580	0.00	0.00	1.05	1.00	1.0	11.00	8.09	1.00	1.06	1.31	1.02
-0.3	2.580	0.00	0.00	1.05	1.00	2.0	2.84	2.23	2.00	1.08	2.61	1.02
-0.3	2.580	0.00	0.00	1.05	1.00	3.0	1.69	1.34	3.01	1.09	3.91	1.02
-0.3	2.580	0.00	0.00	1.05	1.00	4.0	1.23	1.04	4.00	1.07	5.19	1.01
-0.5	2.320	0.00	0.00	1.16	1.00	0.0	364.96	362.32	0.00	1.16	0.00	1.00
-0.5	2.320	0.00	0.00	1.16	1.00	0.5	53.13	30.62	0.50	1.16	0.75	1.01
-0.5	2.320	0.00	0.00	1.16	1.00	1.0	10.08	6.01	1.00	1.17	1.51	1.02
-0.5	2.320	0.00	0.00	1.16	1.00	2.0	2.76	1.85	2.00	1.20	3.02	1.03
-0.5	2.320	0.00	0.00	1.16	1.00	3.0	1.71	1.15	3.01	1.20	4.50	1.02
-0.5	2.320	0.00	0.00	1.16	1.00	4.0	1.28	1.01	4.00	1.19	5.98	1.01
-0.7	2.160	0.00	0.00	1.41	1.00	0.0	364.96	362.32	0.00	1.41	0.00	1.00
-0.7	2.160	0.00	0.00	1.41	1.00	0.5	77.40	22.10	0.50	1.41	0.84	1.01
-0.7	2.160	0.00	0.00	1.41	1.00	1.0	15.09	4.71	1.00	1.41	1.70	1.01
-0.7	2.160	0.00	0.00	1.41	1.00	2.0	3.16	1.60	1.99	1.45	3.41	1.03
-0.7	2.160	0.00	0.00	1.41	1.00	3.0	1.96	1.05	3.01	1.49	5.09	1.01
-0.7	2.160	0.00	0.00	1.41	1.00	4.0	1.50	1.00	4.01	1.46	6.78	1.01

TABLE 13

Residuals of MA(1) Model Transformed to an AR(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAX=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	4.450	-0.01	2.33	0.0000	364.964	-0.0132	2.3319
0.9	4.450	-0.01	2.33	0.5000	7.859	5.1254	2.3845
0.9	4.450	-0.01	2.33	1.0000	1.813	10.2304	2.1815
0.9	4.450	-0.01	2.33	2.0000	1.005	19.7454	2.1191
0.9	4.450	-0.01	2.33	3.0000	1.002	29.7624	2.1199
0.9	4.450	-0.01	2.33	4.0000	1.002	39.7734	2.1201
0.7	4.130	0.00	1.41	0.0000	370.370	-0.0043	1.4137
0.7	4.130	0.00	1.41	0.5000	30.867	1.6599	1.4248
0.7	4.130	0.00	1.41	1.0000	5.204	3.3577	1.4174
0.7	4.130	0.00	1.41	2.0000	1.477	6.6887	1.3761
0.7	4.130	0.00	1.41	3.0000	1.005	9.9117	1.3655
0.7	4.130	0.00	1.41	4.0000	1.001	13.2486	1.3660
0.5	3.750	0.00	1.16	0.0000	373.134	-0.0026	1.1608
0.5	3.750	0.00	1.16	0.5000	46.313	0.9957	1.1609
0.5	3.750	0.00	1.16	1.0000	8.193	2.0246	1.1777
0.5	3.750	0.00	1.16	2.0000	2.022	4.0362	1.1539
0.5	3.750	0.00	1.16	3.0000	1.159	6.0012	1.1698
0.5	3.750	0.00	1.16	4.0000	1.003	7.9480	1.1424
0.3	3.480	0.00	1.05	0.0000	373.134	-0.0018	1.0510
0.3	3.480	0.00	1.05	0.5000	63.857	0.7125	1.0510
0.3	3.480	0.00	1.05	1.0000	11.573	1.4316	1.0607
0.3	3.480	0.00	1.05	2.0000	2.666	2.8608	1.0745
0.3	3.480	0.00	1.05	3.0000	1.474	4.2954	1.0493
0.3	3.480	0.00	1.05	4.0000	1.072	5.7002	1.0518
0.1	3.150	0.00	1.01	0.0000	367.647	-0.0014	1.0062
0.1	3.150	0.00	1.01	0.5000	71.327	0.5541	1.0062
0.1	3.150	0.00	1.01	1.0000	14.131	1.1102	1.0121
0.1	3.150	0.00	1.01	2.0000	3.254	2.2111	1.0253
0.1	3.150	0.00	1.01	3.0000	1.734	3.3593	1.0361
0.1	3.150	0.00	1.01	4.0000	1.217	4.4501	1.0164

TABLE 13

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	2.860	0.00	1.01	0.0000	367.647	-0.0012	1.0059
-0.1	2.860	0.00	1.01	0.5000	81.566	0.4534	1.0059
-0.1	2.860	0.00	1.01	1.0000	16.454	0.9079	1.0091
-0.1	2.860	0.00	1.01	2.0000	3.762	1.8174	1.0260
-0.1	2.860	0.00	1.01	3.0000	1.996	2.7465	1.0308
-0.1	2.860	0.00	1.01	4.0000	1.425	3.6405	1.0153
-0.3	2.580	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.580	0.00	1.05	0.5000	92.251	0.3836	1.0504
-0.3	2.580	0.00	1.05	1.0000	20.458	0.7652	1.0539
-0.3	2.580	0.00	1.05	2.0000	4.417	1.5397	1.0697
-0.3	2.580	0.00	1.05	3.0000	2.332	2.3106	1.0874
-0.3	2.580	0.00	1.05	4.0000	1.649	3.0889	1.0872
-0.5	2.320	0.00	1.16	0.0000	364.964	-0.0009	1.1597
-0.5	2.320	0.00	1.16	0.5000	105.708	0.3325	1.1597
-0.5	2.320	0.00	1.16	1.0000	27.352	0.6656	1.1627
-0.5	2.320	0.00	1.16	2.0000	5.373	1.3376	1.1837
-0.5	2.320	0.00	1.16	3.0000	2.758	1.9992	1.2036
-0.5	2.320	0.00	1.16	4.0000	1.928	2.6859	1.2107
-0.7	2.160	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	2.160	0.00	1.41	0.5000	171.821	0.2933	1.4115
-0.7	2.160	0.00	1.41	1.0000	57.339	0.5800	1.4468
-0.7	2.160	0.00	1.41	2.0000	9.600	1.1789	1.4133
-0.7	2.160	0.00	1.41	3.0000	3.919	1.7645	1.4355
-0.7	2.160	0.00	1.41	4.0000	2.516	2.3549	1.4468

TABLE 14

Residuals of MA(1) Model Transformed to an AR(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAX=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	4.450	-0.01	2.33	0.0000	364.964	-0.0132	2.3319
0.9	4.450	-0.01	2.33	0.5000	1.007	49.7720	2.1213
0.9	4.450	-0.01	2.33	1.0000	1.004	99.8359	2.1218
0.9	4.450	-0.01	2.33	2.0000	1.003	199.9468	2.1314
0.9	4.450	-0.01	2.33	3.0000	1.003	300.0572	2.1457
0.9	4.450	-0.01	2.33	4.0000	1.002	400.1707	2.1651
0.7	4.130	0.00	1.41	0.0000	370.370	-0.0043	1.4137
0.7	4.130	0.00	1.41	0.5000	1.856	5.6390	1.3916
0.7	4.130	0.00	1.41	1.0000	1.004	11.0243	1.3662
0.7	4.130	0.00	1.41	2.0000	1.002	22.1454	1.3665
0.7	4.130	0.00	1.41	3.0000	1.002	33.2688	1.3668
0.7	4.130	0.00	1.41	4.0000	1.002	44.3923	1.3672
0.5	3.750	0.00	1.16	0.0000	373.134	-0.0026	1.1608
0.5	3.750	0.00	1.16	0.5000	8.193	2.0246	1.1777
0.5	3.750	0.00	1.16	1.0000	2.022	4.0362	1.1539
0.5	3.750	0.00	1.16	2.0000	1.004	7.9504	1.1439
0.5	3.750	0.00	1.16	3.0000	1.002	11.9514	1.1426
0.5	3.750	0.00	1.16	4.0000	1.002	15.9558	1.1426
0.3	3.480	0.00	1.05	0.0000	373.134	-0.0018	1.0510
0.3	3.480	0.00	1.05	0.5000	27.404	1.0180	1.0597
0.3	3.480	0.00	1.05	1.0000	5.139	2.0463	1.0642
0.3	3.480	0.00	1.05	2.0000	1.572	4.0957	1.0674
0.3	3.480	0.00	1.05	3.0000	1.035	6.1007	1.0505
0.3	3.480	0.00	1.05	4.0000	1.001	8.1286	1.0489
0.1	3.150	0.00	1.01	0.0000	367.647	-0.0014	1.0062
0.1	3.150	0.00	1.01	0.5000	59.382	0.6159	1.0062
0.1	3.150	0.00	1.01	1.0000	10.773	1.2352	1.0136
0.1	3.150	0.00	1.01	2.0000	2.662	2.4712	1.0331
0.1	3.150	0.00	1.01	3.0000	1.517	3.7212	1.0123
0.1	3.150	0.00	1.01	4.0000	1.108	4.9287	1.0289

TABLE 14

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	2.860	0.00	1.01	0.0000	367.647	-0.0012	1.0059
-0.1	2.860	0.00	1.01	0.5000	95.602	0.4121	1.0059
-0.1	2.860	0.00	1.01	1.0000	20.933	0.8222	1.0116
-0.1	2.860	0.00	1.01	2.0000	4.527	1.6546	1.0205
-0.1	2.860	0.00	1.01	3.0000	2.274	2.4848	1.0314
-0.1	2.860	0.00	1.01	4.0000	1.598	3.3172	1.0332
-0.3	2.580	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.580	0.00	1.05	0.5000	140.449	0.2949	1.0504
-0.3	2.580	0.00	1.05	1.0000	37.571	0.5898	1.0541
-0.3	2.580	0.00	1.05	2.0000	7.309	1.1917	1.0682
-0.3	2.580	0.00	1.05	3.0000	3.510	1.7692	1.0760
-0.3	2.580	0.00	1.05	4.0000	2.249	2.3718	1.0831
-0.5	2.320	0.00	1.16	0.0000	364.964	-0.0009	1.1597
-0.5	2.320	0.00	1.16	0.5000	170.648	0.2213	1.1597
-0.5	2.320	0.00	1.16	1.0000	66.845	0.4436	1.1597
-0.5	2.320	0.00	1.16	2.0000	13.656	0.8902	1.1588
-0.5	2.320	0.00	1.16	3.0000	5.373	1.3376	1.1837
-0.5	2.320	0.00	1.16	4.0000	3.337	1.7700	1.1928
-0.7	2.160	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	2.160	0.00	1.41	0.5000	267.380	0.1722	1.4115
-0.7	2.160	0.00	1.41	1.0000	143.266	0.3452	1.4115
-0.7	2.160	0.00	1.41	2.0000	39.419	0.6907	1.4153
-0.7	2.160	0.00	1.41	3.0000	13.567	1.0393	1.4082
-0.7	2.160	0.00	1.41	4.0000	6.419	1.3888	1.4353

TABLE 15

Residuals of AR(1) Model Transformed to an MA(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAX=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	2.215	0.00	1.35	0.0000	362.319	-0.0002	1.3457
0.9	2.215	0.00	1.35	0.5000	359.712	0.0498	1.3457
0.9	2.215	0.00	1.35	1.0000	340.136	0.0998	1.3457
0.9	2.215	0.00	1.35	2.0000	279.330	0.1998	1.3457
0.9	2.215	0.00	1.35	3.0000	181.818	0.2998	1.3457
0.9	2.215	0.00	1.35	4.0000	109.409	0.3998	1.3457
0.7	2.250	0.00	1.22	0.0000	373.134	-0.0004	1.2210
0.7	2.250	0.00	1.22	0.5000	289.017	0.1496	1.2210
0.7	2.250	0.00	1.22	1.0000	155.763	0.2996	1.2210
0.7	2.250	0.00	1.22	2.0000	37.748	0.5993	1.2255
0.7	2.250	0.00	1.22	3.0000	13.833	0.9000	1.2182
0.7	2.250	0.00	1.22	4.0000	6.672	1.2034	1.2412
0.5	2.350	0.00	1.12	0.0000	370.370	-0.0007	1.1185
0.5	2.350	0.00	1.12	0.5000	162.866	0.2493	1.1185
0.5	2.350	0.00	1.12	1.0000	50.813	0.4993	1.1185
0.5	2.350	0.00	1.12	2.0000	9.440	1.0031	1.1281
0.5	2.350	0.00	1.12	3.0000	4.208	1.4979	1.1450
0.5	2.350	0.00	1.12	4.0000	2.758	1.9996	1.1567
0.3	2.550	0.00	1.04	0.0000	370.370	-0.0009	1.0446
0.3	2.550	0.00	1.04	0.5000	101.010	0.3491	1.0446
0.3	2.550	0.00	1.04	1.0000	24.115	0.6956	1.0471
0.3	2.550	0.00	1.04	2.0000	5.065	1.4026	1.0642
0.3	2.550	0.00	1.04	3.0000	2.558	2.1005	1.0800
0.3	2.550	0.00	1.04	4.0000	1.786	2.8163	1.0800
0.1	2.850	0.00	1.01	0.0000	362.319	-0.0012	1.0057
0.1	2.850	0.00	1.01	0.5000	81.169	0.4488	1.0057
0.1	2.850	0.00	1.01	1.0000	16.670	0.8988	1.0084
0.1	2.850	0.00	1.01	2.0000	3.801	1.8017	1.0264
0.1	2.850	0.00	1.01	3.0000	2.001	2.7181	1.0298
0.1	2.850	0.00	1.01	4.0000	1.438	3.6048	1.0141

TABLE 15

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	3.145	0.00	1.01	0.0000	373.134	-0.0014	1.0060
-0.1	3.145	0.00	1.01	0.5000	72.780	0.5486	1.0060
-0.1	3.145	0.00	1.01	1.0000	14.327	1.0996	1.0110
-0.1	3.145	0.00	1.01	2.0000	3.290	2.1876	1.0250
-0.1	3.145	0.00	1.01	3.0000	1.746	3.3274	1.0354
-0.1	3.145	0.00	1.01	4.0000	1.231	4.4049	1.0137
-0.3	3.380	0.00	1.05	0.0000	373.134	-0.0017	1.0453
-0.3	3.380	0.00	1.05	0.5000	70.323	0.6484	1.0453
-0.3	3.380	0.00	1.05	1.0000	13.105	1.3012	1.0530
-0.3	3.380	0.00	1.05	2.0000	3.019	2.5856	1.0663
-0.3	3.380	0.00	1.05	3.0000	1.586	3.9193	1.0670
-0.3	3.380	0.00	1.05	4.0000	1.117	5.1858	1.0624
-0.5	3.520	0.00	1.12	0.0000	362.319	-0.0019	1.1195
-0.5	3.520	0.00	1.12	0.5000	66.756	0.7481	1.1195
-0.5	3.520	0.00	1.12	1.0000	12.471	1.5011	1.1287
-0.5	3.520	0.00	1.12	2.0000	2.829	2.9913	1.1481
-0.5	3.520	0.00	1.12	3.0000	1.512	4.5227	1.1140
-0.5	3.520	0.00	1.12	4.0000	1.086	5.9909	1.1375
-0.7	3.580	0.00	1.22	0.0000	364.964	-0.0022	1.2224
-0.7	3.580	0.00	1.22	0.5000	63.052	0.8479	1.2224
-0.7	3.580	0.00	1.22	1.0000	11.890	1.7044	1.2367
-0.7	3.580	0.00	1.22	2.0000	2.714	3.3995	1.2494
-0.7	3.580	0.00	1.22	3.0000	1.455	5.1122	1.2126
-0.7	3.580	0.00	1.22	4.0000	1.062	6.7853	1.2164

TABLE 16

Residuals of AR(1) Model Transformed to an MA(1)Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAX=0.0027

LAMBDA= 0.50 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	2.215	0.00	1.35	0.0000	362.319	-0.0002	1.3457
0.9	2.215	0.00	1.35	0.5000	359.712	0.0048	1.3457
0.9	2.215	0.00	1.35	1.0000	362.319	0.0098	1.3457
0.9	2.215	0.00	1.35	2.0000	352.113	0.0198	1.3457
0.9	2.215	0.00	1.35	3.0000	349.650	0.0298	1.3457
0.9	2.215	0.00	1.35	4.0000	354.610	0.0398	1.3457
0.7	2.250	0.00	1.22	0.0000	373.134	-0.0004	1.2210
0.7	2.250	0.00	1.22	0.5000	362.319	0.0446	1.2210
0.7	2.250	0.00	1.22	1.0000	322.581	0.0896	1.2210
0.7	2.250	0.00	1.22	2.0000	248.756	0.1796	1.2210
0.7	2.250	0.00	1.22	3.0000	174.825	0.2696	1.2210
0.7	2.250	0.00	1.22	4.0000	111.607	0.3596	1.2210
0.5	2.350	0.00	1.12	0.0000	370.370	-0.0007	1.1185
0.5	2.350	0.00	1.12	0.5000	280.899	0.1243	1.1185
0.5	2.350	0.00	1.12	1.0000	162.866	0.2493	1.1185
0.5	2.350	0.00	1.12	2.0000	50.813	0.4993	1.1185
0.5	2.350	0.00	1.12	3.0000	18.570	0.7479	1.1205
0.5	2.350	0.00	1.12	4.0000	9.435	1.0029	1.1281
0.3	2.550	0.00	1.04	0.0000	370.370	-0.0009	1.0446
0.3	2.550	0.00	1.04	0.5000	166.667	0.2441	1.0446
0.3	2.550	0.00	1.04	1.0000	52.632	0.4891	1.0446
0.3	2.550	0.00	1.04	2.0000	10.663	0.9812	1.0492
0.3	2.550	0.00	1.04	3.0000	4.650	1.4689	1.0627
0.3	2.550	0.00	1.04	4.0000	2.883	1.9554	1.0793
0.1	2.850	0.00	1.01	0.0000	362.319	-0.0012	1.0057
0.1	2.850	0.00	1.01	0.5000	97.276	0.4038	1.0057
0.1	2.850	0.00	1.01	1.0000	21.779	0.8054	1.0116
0.1	2.850	0.00	1.01	2.0000	4.695	1.6184	1.0205
0.1	2.850	0.00	1.01	3.0000	2.334	2.4332	1.0345
0.1	2.850	0.00	1.01	4.0000	1.620	3.2513	1.0342

TABLE 16

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	3.145	0.00	1.01	0.0000	373.134	-0.0014	1.0060
-0.1	3.145	0.00	1.01	0.5000	61.350	0.6036	1.0060
-0.1	3.145	0.00	1.01	1.0000	11.193	1.2101	1.0129
-0.1	3.145	0.00	1.01	2.0000	2.741	2.4200	1.0334
-0.1	3.145	0.00	1.01	3.0000	1.561	3.6461	1.0254
-0.1	3.145	0.00	1.01	4.0000	1.114	4.8286	1.0271
-0.3	3.380	0.00	1.05	0.0000	373.134	-0.0017	1.0453
-0.3	3.380	0.00	1.05	0.5000	39.174	0.8431	1.0490
-0.3	3.380	0.00	1.05	1.0000	7.129	1.7074	1.0601
-0.3	3.380	0.00	1.05	2.0000	1.919	3.4189	1.0603
-0.3	3.380	0.00	1.05	3.0000	1.144	5.0711	1.0615
-0.3	3.380	0.00	1.05	4.0000	1.003	6.7284	1.0466
-0.5	3.520	0.00	1.12	0.0000	362.319	-0.0019	1.1195
-0.5	3.520	0.00	1.12	0.5000	25.980	1.1183	1.1292
-0.5	3.520	0.00	1.12	1.0000	4.925	2.2581	1.1300
-0.5	3.520	0.00	1.12	2.0000	1.512	4.5227	1.1140
-0.5	3.520	0.00	1.12	3.0000	1.026	6.7250	1.1163
-0.5	3.520	0.00	1.12	4.0000	1.001	8.9642	1.1152
-0.7	3.580	0.00	1.22	0.0000	364.964	-0.0022	1.2224
-0.7	3.580	0.00	1.22	0.5000	18.467	1.4371	1.2345
-0.7	3.580	0.00	1.22	1.0000	3.713	2.8888	1.2407
-0.7	3.580	0.00	1.22	2.0000	1.231	5.7865	1.2225
-0.7	3.580	0.00	1.22	3.0000	1.003	8.6282	1.2134
-0.7	3.580	0.00	1.22	4.0000	1.001	11.5210	1.2141

TABLE 17

MA(1) Model
********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********
***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT
ALPHAY=ALPHAZ=0.0027
LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	2.670	0.00	0.00	1.35	1.00	0.0	364.96	367.65	0.00	1.35	0.00	1.00
0.9	2.670	0.00	0.00	1.35	1.00	0.5	357.14	104.17	0.05	1.35	0.50	1.00
0.9	2.670	0.00	0.00	1.35	1.00	1.0	367.65	24.47	0.10	1.35	0.99	1.01
0.9	2.670	0.00	0.00	1.35	1.00	2.0	352.11	4.11	0.20	1.35	1.99	1.02
0.9	2.670	0.00	0.00	1.35	1.00	3.0	274.73	1.78	0.30	1.35	3.02	1.03
0.9	2.670	0.00	0.00	1.35	1.00	4.0	216.45	1.19	0.40	1.35	4.01	1.01
0.7	2.670	0.00	0.00	1.22	1.00	0.0	362.32	367.65	0.00	1.22	0.00	1.00
0.7	2.670	0.00	0.00	1.22	1.00	0.5	314.47	104.17	0.15	1.22	0.50	1.00
0.7	2.670	0.00	0.00	1.22	1.00	1.0	228.31	24.47	0.30	1.22	0.99	1.01
0.7	2.670	0.00	0.00	1.22	1.00	2.0	97.09	4.11	0.60	1.22	1.99	1.02
0.7	2.670	0.00	0.00	1.22	1.00	3.0	39.70	1.78	0.90	1.23	3.02	1.03
0.7	2.670	0.00	0.00	1.22	1.00	4.0	19.35	1.19	1.20	1.22	4.01	1.01
0.5	2.720	0.00	0.00	1.12	1.00	0.0	370.37	367.65	0.00	1.12	0.00	1.00
0.5	2.720	0.00	0.00	1.12	1.00	0.5	257.73	104.17	0.25	1.12	0.50	1.00
0.5	2.720	0.00	0.00	1.12	1.00	1.0	117.37	24.47	0.50	1.12	0.99	1.01
0.5	2.720	0.00	0.00	1.12	1.00	2.0	26.07	4.11	1.00	1.12	1.99	1.02
0.5	2.720	0.00	0.00	1.12	1.00	3.0	8.11	1.78	1.51	1.14	3.02	1.03
0.5	2.720	0.00	0.00	1.12	1.00	4.0	3.90	1.19	2.00	1.15	4.01	1.01
0.3	2.780	0.00	0.00	1.04	1.00	0.0	362.32	367.65	0.00	1.04	0.00	1.00
0.3	2.780	0.00	0.00	1.04	1.00	0.5	157.23	104.17	0.35	1.04	0.50	1.00
0.3	2.780	0.00	0.00	1.04	1.00	1.0	52.25	24.47	0.70	1.04	0.99	1.01
0.3	2.780	0.00	0.00	1.04	1.00	2.0	8.80	4.11	1.40	1.06	1.99	1.02
0.3	2.780	0.00	0.00	1.04	1.00	3.0	3.33	1.78	2.09	1.07	3.02	1.03
0.3	2.780	0.00	0.00	1.04	1.00	4.0	1.91	1.19	2.82	1.08	4.01	1.01
0.1	2.905	0.00	0.00	1.01	1.00	0.0	373.13	367.65	0.00	1.01	0.00	1.00
0.1	2.905	0.00	0.00	1.01	1.00	0.5	114.16	104.17	0.45	1.01	0.50	1.00
0.1	2.905	0.00	0.00	1.01	1.00	1.0	29.59	24.47	0.90	1.01	0.99	1.01
0.1	2.905	0.00	0.00	1.01	1.00	2.0	4.76	4.11	1.80	1.02	1.99	1.02
0.1	2.905	0.00	0.00	1.01	1.00	3.0	2.05	1.78	2.72	1.03	3.02	1.03
0.1	2.905	0.00	0.00	1.01	1.00	4.0	1.34	1.19	3.61	1.02	4.01	1.01

TABLE 17

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	3.080	0.00	0.00	1.01	1.00	0.0	373.13	367.65	0.00	1.01	0.00	1.00
-0.1	3.080	0.00	0.00	1.01	1.00	0.5	99.21	104.17	0.55	1.01	0.50	1.00
-0.1	3.080	0.00	0.00	1.01	1.00	1.0	21.59	24.47	1.09	1.01	0.99	1.01
-0.1	3.080	0.00	0.00	1.01	1.00	2.0	3.62	4.11	2.20	1.03	1.99	1.02
-0.1	3.080	0.00	0.00	1.01	1.00	3.0	1.59	1.78	3.32	1.03	3.02	1.03
-0.1	3.080	0.00	0.00	1.01	1.00	4.0	1.12	1.19	4.39	1.02	4.01	1.01
-0.3	3.220	0.00	0.00	1.05	1.00	0.0	362.32	367.65	0.00	1.05	0.00	1.00
-0.3	3.220	0.00	0.00	1.05	1.00	0.5	94.52	104.17	0.65	1.05	0.50	1.00
-0.3	3.220	0.00	0.00	1.05	1.00	1.0	18.85	24.47	1.29	1.05	0.99	1.01
-0.3	3.220	0.00	0.00	1.05	1.00	2.0	3.13	4.11	2.58	1.06	1.99	1.02
-0.3	3.220	0.00	0.00	1.05	1.00	3.0	1.38	1.78	3.91	1.05	3.02	1.03
-0.3	3.220	0.00	0.00	1.05	1.00	4.0	1.05	1.19	5.18	1.05	4.01	1.01
-0.5	3.300	0.00	0.00	1.12	1.00	0.0	362.32	367.65	0.00	1.12	0.00	1.00
-0.5	3.300	0.00	0.00	1.12	1.00	0.5	91.74	104.17	0.75	1.12	0.50	1.00
-0.5	3.300	0.00	0.00	1.12	1.00	1.0	17.00	24.47	1.50	1.13	0.99	1.01
-0.5	3.300	0.00	0.00	1.12	1.00	2.0	2.76	4.11	3.00	1.14	1.99	1.02
-0.5	3.300	0.00	0.00	1.12	1.00	3.0	1.26	1.78	4.50	1.12	3.02	1.03
-0.5	3.300	0.00	0.00	1.12	1.00	4.0	1.02	1.19	5.98	1.12	4.01	1.01
-0.7	3.300	0.00	0.00	1.22	1.00	0.0	370.37	367.65	0.00	1.22	0.00	1.00
-0.7	3.300	0.00	0.00	1.22	1.00	0.5	81.97	104.17	0.85	1.22	0.50	1.00
-0.7	3.300	0.00	0.00	1.22	1.00	1.0	15.48	24.47	1.70	1.23	0.99	1.01
-0.7	3.300	0.00	0.00	1.22	1.00	2.0	2.48	4.11	3.41	1.24	1.99	1.02
-0.7	3.300	0.00	0.00	1.22	1.00	3.0	1.21	1.78	5.11	1.23	3.02	1.03
-0.7	3.300	0.00	0.00	1.22	1.00	4.0	1.01	1.19	6.76	1.21	4.01	1.01

TABLE 18

MA(1) Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	2.670	0.00	0.00	1.35	1.00	0.0	364.96	367.65	0.00	1.35	0.00	1.00
0.9	2.670	0.00	0.00	1.35	1.00	0.5	160.77	1.04	0.50	1.35	4.99	1.01
0.9	2.670	0.00	0.00	1.35	1.00	1.0	40.58	1.00	1.00	1.35	9.98	1.01
0.9	2.670	0.00	0.00	1.35	1.00	2.0	5.83	1.00	2.00	1.37	19.99	1.01
0.9	2.670	0.00	0.00	1.35	1.00	3.0	2.46	1.00	3.00	1.40	30.00	1.01
0.9	2.670	0.00	0.00	1.35	1.00	4.0	1.56	1.00	4.00	1.39	40.01	1.01
0.7	2.670	0.00	0.00	1.22	1.00	0.0	362.32	367.65	0.00	1.22	0.00	1.00
0.7	2.670	0.00	0.00	1.22	1.00	0.5	129.20	6.26	0.50	1.22	1.68	1.02
0.7	2.670	0.00	0.00	1.22	1.00	1.0	31.00	1.50	1.00	1.23	3.35	1.01
0.7	2.670	0.00	0.00	1.22	1.00	2.0	4.47	1.00	2.00	1.25	6.64	1.01
0.7	2.670	0.00	0.00	1.22	1.00	3.0	2.07	1.00	3.01	1.27	9.98	1.01
0.7	2.670	0.00	0.00	1.22	1.00	4.0	1.39	1.00	4.00	1.25	13.32	1.01
0.5	2.720	0.00	0.00	1.12	1.00	0.0	370.37	367.65	0.00	1.12	0.00	1.00
0.5	2.720	0.00	0.00	1.12	1.00	0.5	117.37	24.47	0.50	1.12	0.99	1.01
0.5	2.720	0.00	0.00	1.12	1.00	1.0	26.07	4.11	1.00	1.12	1.99	1.02
0.5	2.720	0.00	0.00	1.12	1.00	2.0	3.90	1.19	2.00	1.15	4.01	1.01
0.5	2.720	0.00	0.00	1.12	1.00	3.0	1.85	1.00	3.01	1.15	5.98	1.01
0.5	2.720	0.00	0.00	1.12	1.00	4.0	1.27	1.00	4.00	1.14	7.98	1.01
0.3	2.780	0.00	0.00	1.04	1.00	0.0	362.32	367.65	0.00	1.04	0.00	1.00
0.3	2.780	0.00	0.00	1.04	1.00	0.5	94.34	55.01	0.50	1.04	0.71	1.00
0.3	2.780	0.00	0.00	1.04	1.00	1.0	21.31	9.37	1.00	1.05	1.43	1.01
0.3	2.780	0.00	0.00	1.04	1.00	2.0	3.62	1.91	2.00	1.07	2.89	1.02
0.3	2.780	0.00	0.00	1.04	1.00	3.0	1.71	1.13	3.01	1.08	4.28	1.02
0.3	2.780	0.00	0.00	1.04	1.00	4.0	1.19	1.01	4.01	1.06	5.69	1.01
0.1	2.905	0.00	0.00	1.01	1.00	0.0	373.13	367.65	0.00	1.01	0.00	1.00
0.1	2.905	0.00	0.00	1.01	1.00	0.5	99.21	88.97	0.50	1.01	0.55	1.00
0.1	2.905	0.00	0.00	1.01	1.00	1.0	22.47	18.43	0.99	1.01	1.11	1.01
0.1	2.905	0.00	0.00	1.01	1.00	2.0	3.79	3.27	2.00	1.03	2.21	1.02
0.1	2.905	0.00	0.00	1.01	1.00	3.0	1.72	1.50	3.02	1.04	3.35	1.01
0.1	2.905	0.00	0.00	1.01	1.00	4.0	1.18	1.09	4.01	1.02	4.44	1.03

TABLE 18

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	3.080	0.00	0.00	1.01	1.00	0.0	373.13	367.65	0.00	1.01	0.00	1.00
-0.1	3.080	0.00	0.00	1.01	1.00	0.5	120.19	126.26	0.50	1.01	0.45	1.00
-0.1	3.080	0.00	0.00	1.01	1.00	1.0	27.60	31.50	1.00	1.01	0.91	1.01
-0.1	3.080	0.00	0.00	1.01	1.00	2.0	4.45	5.10	2.00	1.02	1.82	1.02
-0.1	3.080	0.00	0.00	1.01	1.00	3.0	1.87	2.10	3.03	1.03	2.74	1.02
-0.1	3.080	0.00	0.00	1.01	1.00	4.0	1.23	1.34	4.00	1.01	3.64	1.02
-0.3	3.220	0.00	0.00	1.05	1.00	0.0	362.32	367.65	0.00	1.05	0.00	1.00
-0.3	3.220	0.00	0.00	1.05	1.00	0.5	140.45	154.80	0.50	1.05	0.38	1.00
-0.3	3.220	0.00	0.00	1.05	1.00	1.0	37.96	46.84	1.00	1.05	0.77	1.00
-0.3	3.220	0.00	0.00	1.05	1.00	2.0	5.67	7.70	2.01	1.06	1.55	1.02
-0.3	3.220	0.00	0.00	1.05	1.00	3.0	2.20	3.00	3.01	1.06	2.30	1.03
-0.3	3.220	0.00	0.00	1.05	1.00	4.0	1.34	1.70	4.01	1.05	3.10	1.03
-0.5	3.300	0.00	0.00	1.12	1.00	0.0	362.32	367.65	0.00	1.12	0.00	1.00
-0.5	3.300	0.00	0.00	1.12	1.00	0.5	162.87	179.21	0.50	1.12	0.33	1.00
-0.5	3.300	0.00	0.00	1.12	1.00	1.0	49.52	64.02	1.00	1.12	0.67	1.00
-0.5	3.300	0.00	0.00	1.12	1.00	2.0	7.44	11.38	2.02	1.13	1.33	1.01
-0.5	3.300	0.00	0.00	1.12	1.00	3.0	2.78	4.11	2.99	1.15	1.99	1.02
-0.5	3.300	0.00	0.00	1.12	1.00	4.0	1.51	2.18	4.02	1.11	2.68	1.03
-0.7	3.300	0.00	0.00	1.22	1.00	0.0	370.37	367.65	0.00	1.22	0.00	1.00
-0.7	3.300	0.00	0.00	1.22	1.00	0.5	167.22	193.05	0.50	1.22	0.29	1.00
-0.7	3.300	0.00	0.00	1.22	1.00	1.0	58.07	81.30	1.00	1.22	0.59	1.00
-0.7	3.300	0.00	0.00	1.22	1.00	2.0	9.63	15.76	2.01	1.23	1.17	1.00
-0.7	3.300	0.00	0.00	1.22	1.00	3.0	3.43	5.45	2.99	1.24	1.77	1.02
-0.7	3.300	0.00	0.00	1.22	1.00	4.0	1.75	2.85	4.04	1.25	2.35	1.03

TABLE 19

AR(1) Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	3.580	-.01	0.00	2.33	1.00	0.0	370.37	367.65	-0.01	2.33	0.00	1.00
0.9	3.580	-.01	0.00	2.33	1.00	0.5	6.01	104.17	5.05	2.36	0.50	1.00
0.9	3.580	-.01	0.00	2.33	1.00	1.0	1.24	24.47	10.02	2.24	0.99	1.01
0.9	3.580	-.01	0.00	2.33	1.00	2.0	1.00	4.11	19.75	2.12	1.99	1.02
0.9	3.580	-.01	0.00	2.33	1.00	3.0	1.00	1.78	29.76	2.12	3.02	1.03
0.9	3.580	-.01	0.00	2.33	1.00	4.0	1.00	1.19	39.78	2.12	4.01	1.01
0.7	3.515	0.00	0.00	1.41	1.00	0.0	367.65	367.65	0.00	1.41	0.00	1.00
0.7	3.515	0.00	0.00	1.41	1.00	0.5	31.43	104.17	1.66	1.42	0.50	1.00
0.7	3.515	0.00	0.00	1.41	1.00	1.0	4.62	24.47	3.33	1.41	0.99	1.01
0.7	3.515	0.00	0.00	1.41	1.00	2.0	1.12	4.11	6.63	1.41	1.99	1.02
0.7	3.515	0.00	0.00	1.41	1.00	3.0	1.00	1.78	9.91	1.37	3.02	1.03
0.7	3.515	0.00	0.00	1.41	1.00	4.0	1.00	1.19	13.25	1.37	4.01	1.01
0.5	3.400	0.00	0.00	1.16	1.00	0.0	373.13	367.65	0.00	1.16	0.00	1.00
0.5	3.400	0.00	0.00	1.16	1.00	0.5	60.10	104.17	1.00	1.16	0.50	1.00
0.5	3.400	0.00	0.00	1.16	1.00	1.0	9.39	24.47	2.01	1.17	0.99	1.01
0.5	3.400	0.00	0.00	1.16	1.00	2.0	1.69	4.11	4.04	1.17	1.99	1.02
0.5	3.400	0.00	0.00	1.16	1.00	3.0	1.04	1.78	5.98	1.15	3.02	1.03
0.5	3.400	0.00	0.00	1.16	1.00	4.0	1.00	1.19	7.95	1.14	4.01	1.01
0.3	3.250	0.00	0.00	1.05	1.00	0.0	370.37	367.65	0.00	1.05	0.00	1.00
0.3	3.250	0.00	0.00	1.05	1.00	0.5	83.75	104.17	0.71	1.05	0.50	1.00
0.3	3.250	0.00	0.00	1.05	1.00	1.0	15.38	24.47	1.43	1.06	0.99	1.01
0.3	3.250	0.00	0.00	1.05	1.00	2.0	2.55	4.11	2.86	1.07	1.99	1.02
0.3	3.250	0.00	0.00	1.05	1.00	3.0	1.25	1.78	4.29	1.06	3.02	1.03
0.3	3.250	0.00	0.00	1.05	1.00	4.0	1.02	1.19	5.68	1.05	4.01	1.01
0.1	3.080	0.00	0.00	1.01	1.00	0.0	364.96	367.65	0.00	1.01	0.00	1.00
0.1	3.080	0.00	0.00	1.01	1.00	0.5	98.43	104.17	0.55	1.01	0.50	1.00
0.1	3.080	0.00	0.00	1.01	1.00	1.0	20.99	24.47	1.11	1.01	0.99	1.01
0.1	3.080	0.00	0.00	1.01	1.00	2.0	3.57	4.11	2.22	1.03	1.99	1.02
0.1	3.080	0.00	0.00	1.01	1.00	3.0	1.58	1.78	3.35	1.03	3.02	1.03
0.1	3.080	0.00	0.00	1.01	1.00	4.0	1.11	1.19	4.43	1.03	4.01	1.01

TABLE 19

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	2.910	0.00	0.00	1.01	1.00	0.0	370.37	367.65	0.00	1.01	0.00	1.00
-0.1	2.910	0.00	0.00	1.01	1.00	0.5	114.16	104.17	0.45	1.01	0.50	1.00
-0.1	2.910	0.00	0.00	1.01	1.00	1.0	28.78	24.47	0.91	1.01	0.99	1.01
-0.1	2.910	0.00	0.00	1.01	1.00	2.0	4.69	4.11	1.82	1.02	1.99	1.02
-0.1	2.910	0.00	0.00	1.01	1.00	3.0	2.02	1.78	2.74	1.03	3.02	1.03
-0.1	2.910	0.00	0.00	1.01	1.00	4.0	1.31	1.19	3.64	1.02	4.01	1.01
-0.3	2.787	0.00	0.00	1.05	1.00	0.0	362.32	367.65	0.00	1.05	0.00	1.00
-0.3	2.787	0.00	0.00	1.05	1.00	0.5	149.70	104.17	0.38	1.05	0.50	1.00
-0.3	2.787	0.00	0.00	1.05	1.00	1.0	44.76	24.47	0.77	1.05	0.99	1.01
-0.3	2.787	0.00	0.00	1.05	1.00	2.0	6.94	4.11	1.55	1.07	1.99	1.02
-0.3	2.787	0.00	0.00	1.05	1.00	3.0	2.74	1.78	2.31	1.09	3.02	1.03
-0.3	2.787	0.00	0.00	1.05	1.00	4.0	1.68	1.19	3.09	1.09	4.01	1.01
-0.5	2.670	0.00	0.00	1.16	1.00	0.0	367.65	367.65	0.00	1.16	0.00	1.00
-0.5	2.670	0.00	0.00	1.16	1.00	0.5	183.15	104.17	0.33	1.16	0.50	1.00
-0.5	2.670	0.00	0.00	1.16	1.00	1.0	69.06	24.47	0.67	1.16	0.99	1.01
-0.5	2.670	0.00	0.00	1.16	1.00	2.0	12:47	4.11	1.33	1.16	1.99	1.02
-0.5	2.670	0.00	0.00	1.16	1.00	3.0	4.01	1.78	2.00	1.19	3.02	1.03
-0.5	2.670	0.00	0.00	1.16	1.00	4.0	2.26	1.19	2.67	1.21	4.01	1.01
-0.7	2.570	0.00	0.00	1.41	1.00	0.0	364.96	367.65	0.00	1.41	0.00	1.00
-0.7	2.570	0.00	0.00	1.41	1.00	0.5	257.73	104.17	0.29	1.41	0.50	1.00
-0.7	2.570	0.00	0.00	1.41	1.00	1.0	120.48	24.47	0.59	1.41	0.99	1.01
-0.7	2.570	0.00	0.00	1.41	1.00	2.0	28.06	4.11	1.18	1.41	1.99	1.02
-0.7	2.570	0.00	0.00	1.41	1.00	3.0	8.60	1.78	1.77	1.43	3.02	1.03
-0.7	2.570	0.00	0.00	1.41	1.00	4.0	3.73	1.19	2.35	1.45	4.01	1.01

TABLE 20

AR(1) Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAY=ALPHAZ=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
0.9	3.580	-.01	0.00	2.33	1.00	0.0	370.37	367.65	-0.01	2.33	0.00	1.00
0.9	3.580	-.01	0.00	2.33	1.00	0.5	308.64	357.14	0.49	2.33	0.05	1.00
0.9	3.580	-.01	0.00	2.33	1.00	1.0	197.63	331.13	0.99	2.33	0.10	1.00
0.9	3.580	-.01	0.00	2.33	1.00	2.0	69.16	257.73	1.99	2.33	0.20	1.00
0.9	3.580	-.01	0.00	2.33	1.00	3.0	25.51	190.11	2.95	2.39	0.30	1.00
0.9	3.580	-.01	0.00	2.33	1.00	4.0	11.66	147.06	4.03	2.37	0.40	1.00
0.7	3.515	0.00	0.00	1.41	1.00	0.0	367.65	367.65	0.00	1.41	0.00	1.00
0.7	3.515	0.00	0.00	1.41	1.00	0.5	236.97	290.70	0.50	1.41	0.15	1.00
0.7	3.515	0.00	0.00	1.41	1.00	1.0	93.63	190.11	1.00	1.41	0.30	1.00
0.7	3.515	0.00	0.00	1.41	1.00	2.0	19.47	78.99	1.99	1.43	0.60	1.00
0.7	3.515	0.00	0.00	1.41	1.00	3.0	6.01	32.13	3.02	1.42	0.90	1.01
0.7	3.515	0.00	0.00	1.41	1.00	4.0	2.84	15.07	3.98	1.42	1.20	1.00
0.5	3.400	0.00	0.00	1.16	1.00	0.0	373.13	367.65	0.00	1.16	0.00	1.00
0.5	3.400	0.00	0.00	1.16	1.00	0.5	176.68	220.26	0.50	1.16	0.25	1.00
0.5	3.400	0.00	0.00	1.16	1.00	1.0	60.10	104.17	1.00	1.16	0.50	1.00
0.5	3.400	0.00	0.00	1.16	1.00	2.0	9.39	24.47	2.01	1.17	0.99	1.01
0.5	3.400	0.00	0.00	1.16	1.00	3.0	3.33	8.23	2.98	1.17	1.51	1.02
0.5	3.400	0.00	0.00	1.16	1.00	4.0	1.69	4.11	4.04	1.17	1.99	1.02
0.3	3.250	0.00	0.00	1.05	1.00	0.0	370.37	367.65	0.00	1.05	0.00	1.00
0.3	3.250	0.00	0.00	1.05	1.00	0.5	148.37	169.49	0.50	1.05	0.35	1.00
0.3	3.250	0.00	0.00	1.05	1.00	1.0	40.35	57.34	1.00	1.05	0.70	1.00
0.3	3.250	0.00	0.00	1.05	1.00	2.0	6.00	10.03	2.01	1.06	1.40	1.01
0.3	3.250	0.00	0.00	1.05	1.00	3.0	2.31	3.63	3.01	1.06	2.10	1.02
0.3	3.250	0.00	0.00	1.05	1.00	4.0	1.35	1.99	4.01	1.06	2.82	1.02
0.1	3.080	0.00	0.00	1.01	1.00	0.0	364.96	367.65	0.00	1.01	0.00	1.00
0.1	3.080	0.00	0.00	1.01	1.00	0.5	120.48	129.20	0.50	1.01	0.45	1.00
0.1	3.080	0.00	0.00	1.01	1.00	1.0	27.58	32.13	1.00	1.01	0.90	1.01
0.1	3.080	0.00	0.00	1.01	1.00	2.0	4.43	5.18	2.00	1.02	1.81	1.02
0.1	3.080	0.00	0.00	1.01	1.00	3.0	1.87	2.13	3.03	1.03	2.71	1.03
0.1	3.080	0.00	0.00	1.01	1.00	4.0	1.23	1.35	4.00	1.01	3.61	1.01

TABLE 20

THET	Contrl Limit	Av.Y	Av.Z	Sd.Y	Sd.Z	SHIFT	ARLY	ARLZ	Av.Y	Av.Z	Sd.Y	Sd.Z
		BEFORE SHIFT HAS OCCURED							***AFTER THE SHIFT HAS OCCURED***			
-0.1	2.910	0.00	0.00	1.01	1.00	0.0	370.37	367.65	0.00	1.01	0.00	1.00
-0.1	2.910	0.00	0.00	1.01	1.00	0.5	101.21	91.24	0.50	1.01	0.55	1.00
-0.1	2.910	0.00	0.00	1.01	1.00	1.0	22.69	19.15	0.99	1.01	1.09	1.01
-0.1	2.910	0.00	0.00	1.01	1.00	2.0	3.79	3.33	2.00	1.03	2.19	1.02
-0.1	2.910	0.00	0.00	1.01	1.00	3.0	1.73	1.52	3.02	1.04	3.32	1.01
-0.1	2.910	0.00	0.00	1.01	1.00	4.0	1.18	1.11	4.01	1.02	4.39	1.02
-0.3	2.787	0.00	0.00	1.05	1.00	0.0	362.32	367.65	0.00	1.05	0.00	1.00
-0.3	2.787	0.00	0.00	1.05	1.00	0.5	97.47	66.84	0.50	1.05	0.65	1.00
-0.3	2.787	0.00	0.00	1.05	1.00	1.0	22.79	12.26	0.99	1.06	1.30	1.01
-0.3	2.787	0.00	0.00	1.05	1.00	2.0	3.68	2.31	2.00	1.08	2.60	1.02
-0.3	2.787	0.00	0.00	1.05	1.00	3.0	1.75	1.22	3.02	1.09	3.91	1.01
-0.3	2.787	0.00	0.00	1.05	1.00	4.0	1.20	1.02	4.01	1.07	5.18	1.02
-0.5	2.670	0.00	0.00	1.16	1.00	0.0	367.65	367.65	0.00	1.16	0.00	1.00
-0.5	2.670	0.00	0.00	1.16	1.00	0.5	113.64	49.07	0.50	1.16	0.75	1.00
-0.5	2.670	0.00	0.00	1.16	1.00	1.0	26.66	8.24	1.00	1.16	1.51	1.02
-0.5	2.670	0.00	0.00	1.16	1.00	2.0	4.01	1.78	2.00	1.19	3.02	1.03
-0.5	2.670	0.00	0.00	1.16	1.00	3.0	1.89	1.09	3.02	1.21	4.49	1.03
-0.5	2.670	0.00	0.00	1.16	1.00	4.0	1.31	1.00	4.01	1.20	5.98	1.01
-0.7	2.570	0.00	0.00	1.41	1.00	0.0	364.96	367.65	0.00	1.41	0.00	1.00
-0.7	2.570	0.00	0.00	1.41	1.00	0.5	149.70	36.55	0.50	1.41	0.85	1.01
-0.7	2.570	0.00	0.00	1.41	1.00	1.0	42.87	5.99	1.00	1.42	1.71	1.01
-0.7	2.570	0.00	0.00	1.41	1.00	2.0	5.88	1.46	2.00	1.44	3.41	1.01
-0.7	2.570	0.00	0.00	1.41	1.00	3.0	2.35	1.03	3.00	1.48	5.08	1.02
-0.7	2.570	0.00	0.00	1.41	1.00	4.0	1.63	1.00	4.01	1.47	6.78	1.01

TABLE 21

Residuals of MA(1) Model Transformed to an AR(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAX=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	3.580	-0.01	2.33	0.0000	370.370	-0.0132	2.3319
0.9	3.580	-0.01	2.33	0.5000	6.013	5.0527	2.3588
0.9	3.580	-0.01	2.33	1.0000	1.238	10.0202	2.2362
0.9	3.580	-0.01	2.33	2.0000	1.003	19.7484	2.1202
0.9	3.580	-0.01	2.33	3.0000	1.002	29.7624	2.1199
0.9	3.580	-0.01	2.33	4.0000	1.001	39.7784	2.1161
0.7	3.515	0.00	1.41	0.0000	367.647	-0.0043	1.4137
0.7	3.515	0.00	1.41	0.5000	31.431	1.6609	1.4236
0.7	3.515	0.00	1.41	1.0000	4.621	3.3312	1.4068
0.7	3.515	0.00	1.41	2.0000	1.121	6.6251	1.4075
0.7	3.515	0.00	1.41	3.0000	1.001	9.9116	1.3659
0.7	3.515	0.00	1.41	4.0000	1.001	13.2486	1.3660
0.5	3.400	0.00	1.16	0.0000	373.134	-0.0026	1.1608
0.5	3.400	0.00	1.16	0.5000	60.096	0.9975	1.1608
0.5	3.400	0.00	1.16	1.0000	9.390	2.0111	1.1748
0.5	3.400	0.00	1.16	2.0000	1.693	4.0358	1.1698
0.5	3.400	0.00	1.16	3.0000	1.043	5.9759	1.1468
0.5	3.400	0.00	1.16	4.0000	1.001	7.9481	1.1427
0.3	3.250	0.00	1.05	0.0000	370.370	-0.0018	1.0510
0.3	3.250	0.00	1.05	0.5000	83.752	0.7125	1.0510
0.3	3.250	0.00	1.05	1.0000	15.380	1.4253	1.0585
0.3	3.250	0.00	1.05	2.0000	2.551	2.8571	1.0714
0.3	3.250	0.00	1.05	3.0000	1.249	4.2865	1.0581
0.3	3.250	0.00	1.05	4.0000	1.021	5.6842	1.0485
0.1	3.080	0.00	1.01	0.0000	364.964	-0.0014	1.0062
0.1	3.080	0.00	1.01	0.5000	98.425	0.5541	1.0062
0.1	3.080	0.00	1.01	1.0000	20.987	1.1060	1.0145
0.1	3.080	0.00	1.01	2.0000	3.570	2.2221	1.0287
0.1	3.080	0.00	1.01	3.0000	1.575	3.3466	1.0291
0.1	3.080	0.00	1.01	4.0000	1.114	4.4325	1.0273

TABLE 21

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	2.910	0.00	1.01	0.0000	370.370	-0.0012	1.0059
-0.1	2.910	0.00	1.01	0.5000	114.155	0.4534	1.0059
-0.1	2.910	0.00	1.01	1.0000	28.779	0.9092	1.0131
-0.1	2.910	0.00	1.01	2.0000	4.689	1.8166	1.0204
-0.1	2.910	0.00	1.01	3.0000	2.018	2.7445	1.0295
-0.1	2.910	0.00	1.01	4.0000	1.313	3.6392	1.0203
-0.3	2.787	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.787	0.00	1.05	0.5000	149.701	0.3836	1.0504
-0.3	2.787	0.00	1.05	1.0000	44.761	0.7681	1.0522
-0.3	2.787	0.00	1.05	2.0000	6.938	1.5495	1.0690
-0.3	2.787	0.00	1.05	3.0000	2.740	2.3077	1.0861
-0.3	2.787	0.00	1.05	4.0000	1.681	3.0900	1.0868
-0.5	2.670	0.00	1.16	0.0000	367.647	-0.0009	1.1597
-0.5	2.670	0.00	1.16	0.5000	183.150	0.3325	1.1597
-0.5	2.670	0.00	1.16	1.0000	69.061	0.6658	1.1597
-0.5	2.670	0.00	1.16	2.0000	12.465	1.3337	1.1597
-0.5	2.670	0.00	1.16	3.0000	4.009	1.9973	1.1872
-0.5	2.670	0.00	1.16	4.0000	2.257	2.6714	1.2067
-0.7	2.570	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	2.570	0.00	1.41	0.5000	257.732	0.2933	1.4115
-0.7	2.570	0.00	1.41	1.0000	120.482	0.5875	1.4115
-0.7	2.570	0.00	1.41	2.0000	28.060	1.1761	1.4124
-0.7	2.570	0.00	1.41	3.0000	8.601	1.7695	1.4348
-0.7	2.570	0.00	1.41	4.0000	3.728	2.3524	1.4476

TABLE 22

Residuals of MA(1) Model Transformed to an AR(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAX=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	3.580	-0.01	2.33	0.0000	370.370	-0.0132	2.3319
0.9	3.580	-0.01	2.33	0.5000	1.006	49.7754	2.1202
0.9	3.580	-0.01	2.33	1.0000	1.004	99.8359	2.1218
0.9	3.580	-0.01	2.33	2.0000	1.003	199.9468	2.1314
0.9	3.580	-0.01	2.33	3.0000	1.002	300.0604	2.1453
0.9	3.580	-0.01	2.33	4.0000	1.002	400.1707	2.1651
0.7	3.515	0.00	1.41	0.0000	367.647	-0.0043	1.4137
0.7	3.515	0.00	1.41	0.5000	1.403	5.5757	1.3840
0.7	3.515	0.00	1.41	1.0000	1.003	11.0223	1.3656
0.7	3.515	0.00	1.41	2.0000	1.002	22.1454	1.3665
0.7	3.515	0.00	1.41	3.0000	1.002	33.2688	1.3668
0.7	3.515	0.00	1.41	4.0000	1.002	44.3923	1.3672
0.5	3.400	0.00	1.16	0.0000	373.134	-0.0026	1.1608
0.5	3.400	0.00	1.16	0.5000	9.390	2.0111	1.1748
0.5	3.400	0.00	1.16	1.0000	1.695	4.0370	1.1696
0.5	3.400	0.00	1.16	2.0000	1.002	7.9470	1.1425
0.5	3.400	0.00	1.16	3.0000	1.002	11.9514	1.1426
0.5	3.400	0.00	1.16	4.0000	1.001	15.9570	1.1428
0.3	3.250	0.00	1.05	0.0000	370.370	-0.0018	1.0510
0.3	3.250	0.00	1.05	0.5000	38.056	1.0172	1.0552
0.3	3.250	0.00	1.05	1.0000	5.708	2.0473	1.0645
0.3	3.250	0.00	1.05	2.0000	1.323	4.0908	1.0578
0.3	3.250	0.00	1.05	3.0000	1.008	6.0847	1.0496
0.3	3.250	0.00	1.05	4.0000	1.001	8.1286	1.0489
0.1	3.080	0.00	1.01	0.0000	364.964	-0.0014	1.0062
0.1	3.080	0.00	1.01	0.5000	83.893	0.6159	1.0062
0.1	3.080	0.00	1.01	1.0000	15.770	1.2323	1.0111
0.1	3.080	0.00	1.01	2.0000	2.854	2.4644	1.0353
0.1	3.080	0.00	1.01	3.0000	1.351	3.7147	1.0187
0.1	3.080	0.00	1.01	4.0000	1.045	4.9296	1.0143

TABLE 22

THET	Contrl Limit	Av.X	Sd.X	SHIFT	ARLX	Av.X	Sd.X
		***BEFORE	SHIFT***			****AFTER	SHIFT****
-0.1	2.910	0.00	1.01	0.0000	370.370	-0.0012	1.0059
-0.1	2.910	0.00	1.01	0.5000	132.626	0.4121	1.0059
-0.1	2.910	0.00	1.01	1.0000	36.581	0.8245	1.0103
-0.1	2.910	0.00	1.01	2.0000	6.011	1.6579	1.0213
-0.1	2.910	0.00	1.01	3.0000	2.379	2.4883	1.0342
-0.1	2.910	0.00	1.01	4.0000	1.489	3.3161	1.0195
-0.3	2.787	0.00	1.05	0.0000	362.319	-0.0010	1.0504
-0.3	2.787	0.00	1.05	0.5000	199.203	0.2949	1.0504
-0.3	2.787	0.00	1.05	1.0000	73.529	0.5907	1.0504
-0.3	2.787	0.00	1.05	2.0000	14.325	1.1831	1.0503
-0.3	2.787	0.00	1.05	3.0000	4.859	1.7770	1.0708
-0.3	2.787	0.00	1.05	4.0000	2.609	2.3700	1.0834
-0.5	2.670	0.00	1.16	0.0000	367.647	-0.0009	1.1597
-0.5	2.670	0.00	1.16	0.5000	270.270	0.2213	1.1597
-0.5	2.670	0.00	1.16	1.0000	138.504	0.4436	1.1597
-0.5	2.670	0.00	1.16	2.0000	36.181	0.8876	1.1629
-0.5	2.670	0.00	1.16	3.0000	12.465	1.3337	1.1597
-0.5	2.670	0.00	1.16	4.0000	5.359	1.7820	1.1844
-0.7	2.570	0.00	1.41	0.0000	364.964	-0.0008	1.4115
-0.7	2.570	0.00	1.41	0.5000	316.456	0.1722	1.4115
-0.7	2.570	0.00	1.41	1.0000	229.358	0.3452	1.4115
-0.7	2.570	0.00	1.41	2.0000	92.764	0.6913	1.4115
-0.7	2.570	0.00	1.41	3.0000	39.255	1.0370	1.4153
-0.7	2.570	0.00	1.41	4.0000	18.110	1.3817	1.4122

TABLE 23

Residuals of AR(1) Model Transformed to an MA(1)Model

*********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE INPUT

ALPHAX=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
0.9	2.670	0.00	1.35	0.0000	364.964	-0.0002	1.3457
0.9	2.670	0.00	1.35	0.5000	357.143	0.0498	1.3457
0.9	2.670	0.00	1.35	1.0000	367.647	0.0998	1.3457
0.9	2.670	0.00	1.35	2.0000	352.113	0.1998	1.3457
0.9	2.670	0.00	1.35	3.0000	274.725	0.2998	1.3457
0.9	2.670	0.00	1.35	4.0000	216.450	0.3998	1.3457
0.7	2.670	0.00	1.22	0.0000	362.319	-0.0004	1.2210
0.7	2.670	0.00	1.22	0.5000	314.465	0.1496	1.2210
0.7	2.670	0.00	1.22	1.0000	228.311	0.2996	1.2210
0.7	2.670	0.00	1.22	2.0000	97.087	0.5996	1.2210
0.7	2.670	0.00	1.22	3.0000	39.700	0.8993	1.2264
0.7	2.670	0.00	1.22	4.0000	19.352	1.1986	1.2217
0.5	2.720	0.00	1.12	0.0000	370.370	-0.0007	1.1185
0.5	2.720	0.00	1.12	0.5000	257.732	0.2493	1.1185
0.5	2.720	0.00	1.12	1.0000	117.371	0.4993	1.1185
0.5	2.720	0.00	1.12	2.0000	26.074	0.9979	1.1200
0.5	2.720	0.00	1.12	3.0000	8.107	1.5070	1.1353
0.5	2.720	0.00	1.12	4.0000	3.905	1.9995	1.1468
0.3	2.780	0.00	1.04	0.0000	362.319	-0.0009	1.0446
0.3	2.780	0.00	1.04	0.5000	157.233	0.3491	1.0446
0.3	2.780	0.00	1.04	1.0000	52.247	0.6991	1.0446
0.3	2.780	0.00	1.04	2.0000	8.802	1.4042	1.0576
0.3	2.780	0.00	1.04	3.0000	3.334	2.0924	1.0699
0.3	2.780	0.00	1.04	4.0000	1.907	2.8210	1.0787
0.1	2.905	0.00	1.01	0.0000	373.134	-0.0012	1.0057
0.1	2.905	0.00	1.01	0.5000	114.155	0.4488	1.0057
0.1	2.905	0.00	1.01	1.0000	29.593	0.8986	1.0119
0.1	2.905	0.00	1.01	2.0000	4.760	1.8015	1.0226
0.1	2.905	0.00	1.01	3.0000	2.045	2.7156	1.0283
0.1	2.905	0.00	1.01	4.0000	1.335	3.6059	1.0247

TABLE 23

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	3.080	0.00	1.01	0.0000	373.134	-0.0014	1.0060
-0.1	3.080	0.00	1.01	0.5000	99.206	0.5486	1.0060
-0.1	3.080	0.00	1.01	1.0000	21.589	1.0948	1.0134
-0.1	3.080	0.00	1.01	2.0000	3.623	2.2032	1.0265
-0.1	3.080	0.00	1.01	3.0000	1.594	3.3157	1.0303
-0.1	3.080	0.00	1.01	4.0000	1.122	4.3881	1.0250
-0.3	3.220	0.00	1.05	0.0000	362.319	-0.0017	1.0453
-0.3	3.220	0.00	1.05	0.5000	94.518	0.6484	1.0453
-0.3	3.220	0.00	1.05	1.0000	18.846	1.2935	1.0528
-0.3	3.220	0.00	1.05	2.0000	3.128	2.5850	1.0645
-0.3	3.220	0.00	1.05	3.0000	1.384	3.9090	1.0486
-0.3	3.220	0.00	1.05	4.0000	1.055	5.1835	1.0461
-0.5	3.300	0.00	1.12	0.0000	362.319	-0.0019	1.1195
-0.5	3.300	0.00	1.12	0.5000	91.743	0.7481	1.1195
-0.5	3.300	0.00	1.12	1.0000	16.997	1.4956	1.1287
-0.5	3.300	0.00	1.12	2.0000	2.759	2.9984	1.1445
-0.5	3.300	0.00	1.12	3.0000	1.265	4.5008	1.1212
-0.5	3.300	0.00	1.12	4.0000	1.025	5.9751	1.1165
-0.7	3.300	0.00	1.22	0.0000	370.370	-0.0022	1.2224
-0.7	3.300	0.00	1.22	0.5000	81.967	0.8479	1.2224
-0.7	3.300	0.00	1.22	1.0000	15.483	1.6967	1.2326
-0.7	3.300	0.00	1.22	2.0000	2.485	3.4053	1.2416
-0.7	3.300	0.00	1.22	3.0000	1.209	5.1079	1.2289
-0.7	3.300	0.00	1.22	4.0000	1.015	6.7575	1.2121

TABLE 22

Residuals of AR(1) Model Transformed to an MA(1)Model

********** CONTROL LIMIT FACTOR FOR Zs = 2.99**********

***** SHIFT IN MEAN DUE TO A STEP CHANGE IN THE OUTPUT

ALPHAX=0.0027

LAMBDA= 0.75 GEOMETRIC MOVING AVERAGE CHART

THET	Contrl Limit	Av.X ***BEFORE	SHIFT*** Sd.X	SHIFT	ARLX	Av.X ****AFTER	SHIFT**** Sd.X
0.9	2.670	0.00	1.35	0.0000	364.964	-0.0002	1.3457
0.9	2.670	0.00	1.35	0.5000	364.964	0.0048	1.3457
0.9	2.670	0.00	1.35	1.0000	362.319	0.0098	1.3457
0.9	2.670	0.00	1.35	2.0000	359.712	0.0198	1.3457
0.9	2.670	0.00	1.35	3.0000	373.134	0.0298	1.3457
0.9	2.670	0.00	1.35	4.0000	362.319	0.0398	1.3457
0.7	2.670	0.00	1.22	0.0000	362.319	-0.0004	1.2210
0.7	2.670	0.00	1.22	0.5000	357.143	0.0446	1.2210
0.7	2.670	0.00	1.22	1.0000	347.222	0.0896	1.2210
0.7	2.670	0.00	1.22	2.0000	312.500	0.1796	1.2210
0.7	2.670	0.00	1.22	3.0000	257.732	0.2696	1.2210
0.7	2.670	0.00	1.22	4.0000	205.761	0.3596	1.2210
0.5	2.720	0.00	1.12	0.0000	370.370	-0.0007	1.1185
0.5	2.720	0.00	1.12	0.5000	322.581	0.1243	1.1185
0.5	2.720	0.00	1.12	1.0000	257.732	0.2493	1.1185
0.5	2.720	0.00	1.12	2.0000	117.371	0.4993	1.1185
0.5	2.720	0.00	1.12	3.0000	52.356	0.7494	1.1185
0.5	2.720	0.00	1.12	4.0000	26.074	0.9979	1.1200
0.3	2.780	0.00	1.04	0.0000	362.319	-0.0009	1.0446
0.3	2.780	0.00	1.04	0.5000	233.645	0.2441	1.0446
0.3	2.780	0.00	1.04	1.0000	98.425	0.4891	1.0446
0.3	2.780	0.00	1.04	2.0000	22.906	0.9741	1.0492
0.3	2.780	0.00	1.04	3.0000	7.617	1.4791	1.0633
0.3	2.780	0.00	1.04	4.0000	3.747	1.9588	1.0687
0.1	2.905	0.00	1.01	0.0000	373.134	-0.0012	1.0057
0.1	2.905	0.00	1.01	0.5000	133.333	0.4038	1.0057
0.1	2.905	0.00	1.01	1.0000	37.957	0.8077	1.0093
0.1	2.905	0.00	1.01	2.0000	6.211	1.6272	1.0207
0.1	2.905	0.00	1.01	3.0000	2.474	2.4327	1.0344
0.1	2.905	0.00	1.01	4.0000	1.516	3.2546	1.0190

TABLE 22

THET	Contrl Limit	Av.X ***BEFORE	Sd.X SHIFT***	SHIFT	ARLX	Av.X ****AFTER	Sd.X SHIFT****
-0.1	3.080	0.00	1.01	0.0000	373.134	-0.0014	1.0060
-0.1	3.080	0.00	1.01	0.5000	86.655	0.6036	1.0060
-0.1	3.080	0.00	1.01	1.0000	16.921	1.2074	1.0108
-0.1	3.080	0.00	1.01	2.0000	2.979	2.4086	1.0301
-0.1	3.080	0.00	1.01	3.0000	1.380	3.6370	1.0155
-0.1	3.080	0.00	1.01	4.0000	1.053	4.8274	1.0126
-0.3	3.220	0.00	1.05	0.0000	362.319	-0.0017	1.0453
-0.3	3.220	0.00	1.05	0.5000	56.689	0.8433	1.0453
-0.3	3.220	0.00	1.05	1.0000	9.013	1.6983	1.0576
-0.3	3.220	0.00	1.05	2.0000	1.715	3.4057	1.0679
-0.3	3.220	0.00	1.05	3.0000	1.069	5.0599	1.0468
-0.3	3.220	0.00	1.05	4.0000	1.001	6.7277	1.0467
-0.5	3.300	0.00	1.12	0.0000	362.319	-0.0019	1.1195
-0.5	3.300	0.00	1.12	0.5000	37.368	1.1215	1.1239
-0.5	3.300	0.00	1.12	1.0000	5.565	2.2618	1.1352
-0.5	3.300	0.00	1.12	2.0000	1.266	4.5008	1.1207
-0.5	3.300	0.00	1.12	3.0000	1.004	6.7148	1.1170
-0.5	3.300	0.00	1.12	4.0000	1.001	8.9642	1.1152
-0.7	3.300	0.00	1.22	0.0000	370.370	-0.0022	1.2224
-0.7	3.300	0.00	1.22	0.5000	23.641	1.4326	1.2359
-0.7	3.300	0.00	1.22	1.0000	3.736	2.8885	1.2396
-0.7	3.300	0.00	1.22	2.0000	1.088	5.7698	1.2412
-0.7	3.300	0.00	1.22	3.0000	1.002	8.6270	1.2136
-0.7	3.300	0.00	1.22	4.0000	1.001	11.5210	1.2141

VITA

Ramesh Ananthakrishnan was born September 8, 1965 to Mani and Vijayalakshmi Ananthakrishnan in Coimbatore, India. He graduated from Regional Engineering College, Tiruchirapalli with a B.E. in Production Engineering in June of 1987. He worked with Lucas-TVS, Madras, India for a year as a Manufacturing Engineer prior to joining Lehigh University.